Sub-Riemannian geodesic flow for Goursat distribution

Sergey Agapov
Sobolev Institute of Mathematics, Novosibirsk, Russia

The following optimal control problem is considered:

$$\dot{q} = u_1 f_1(q) + u_2 f_2(q), \qquad q = (x_1, x_2, ..., x_n) \in \mathbb{R}^n, \qquad u \in \mathbb{R}^2, \tag{1}$$

where

$$f_1(q) = (1, 0, -x_2, \ldots, -x_{n-1}),$$
$$f_2(q) = (0, 1, 0, \ldots, 0)$$

are vector fields defining the distribution of two-dimensional planes in $\mathbb{R}^n$ (the so-called Goursat distribution), u is a control parameter. Boundary conditions: $q(0) = q_0,\ q(t_1) = q_1$. The functional to be minimized is as follows:

$$L = \int_0^{t_1} \sqrt{u_1^2 + u_2^2} dt.$$

The system (1) is completely controllable and optimal trajectories exist (see [1]). Via Pontryagin's maximum principle (see [1]) we obtain the corresponding Hamiltonian system which is proved to be completely integrable (in the Liouville sense), all the first integrals being found explicitly. The level surfaces of these integrals are described. Finally, we study motion-planning problem related to the Goursat distribution. Namely, we search for the trajectories which are periodic in some coordinates. They are related to the motion along the "prohibited" directions. This problem plays an important role in applications (for example, in robotics, see [2], [3]).

References

[1] A.A. Agrachev, Yu.L. Sachkov, Geometrical control theory. *FizMatLit 2005.*

[2] J.P. Laumond, Robot Motion Planning and Control, Springer, Berlin, Heidelberg, 1998.

[3] N.B. Mel'nikov, Optimality of singular curves in the problem of a car with n trailers , Optimal control, SMNF, 19, 2006, 114–130.