On subgraphs of graph of binary relations

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Any binary relation $R \subseteq X^2$ (where X is arbitrary set) generates on the set X^2 characteristic function: if $(x, y) \in R$, then $R\,(x, y) = 1$, otherwise $R\,(x, y) = 0$. In terms of characteristic functions on the set of all binary relations of the set X we introduced the concept of a binary reflexive relation of adjacency [1,2] and determined the algebraic system consisting of all binary relations of set and of all unordered pairs various adjacent binary relations. If X is finite set then this algebraic system is the graph («the graph of graphs»). We investigated some its subgraphs.

The following proposition hold. Let σ and τ are adjacent relations, then 1) σ is a partial order if and only if τ is a partial order; 2) σ is a reflexive-transitive relation if and only if τ is a reflexive-transitive relation; 3) σ is an acyclic relation (acyclic digraph) if and only if τ is a acyclic relation (acyclic digraph).

We investigated some features of the structure of the graph of partial orders, the graph of reflexive-transitive relations and the graph of acyclic relations.

In particular, if X consists of n elements, and $T_0(n)$ is the number of labeled T_0-topologies defined on the set X, then the number of vertices in a graph of partial orders is $T_0(n)$, and the number of connected components is $T_0(n-1)$. Similarly in a graph of reflexive-transitive relations the number of connected components equal

$$\sum_{m=1}^{n} S(n,m)\,T_0(m-1),$$

where $S(n,m)$ is Stirling number of second kind. It is well known (see for example [3]) that the number of vertices in a graph equal

$$\sum_{m=1}^{n} S(n,m)\,T_0(m).$$

In a graph of acyclic relations the number of connected components equal

$$\sum_{p_1+\ldots+p_k=n} \frac{(-1)^{n-k}}{k}\,\frac{n!}{p_1!\ldots p_k!}\,2^{\,(n^2-p_1^2-\ldots-p_k^2)/2}.$$

According to [4] the number of vertices in a graph equal

$$\sum_{p_1+\ldots+p_k=n} (-1)^{n-k}\,\frac{n!}{p_1!\ldots p_k!}\,2^{\,(n^2-p_1^2-\ldots-p_k^2)/2}.$$

References

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