Central Unit Group of Integral Group Ring of $GL(2,4)$

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The groups $GL(2,q)$ $(q>2)$ have nontrivial centers. This reason is the source of certain difficulties of finding central unit groups of integral group ring of those groups. In [1] there is the complete description of central unit group of integral group ring of $GL(2,5)$.

Note that $GL(2,4) = Z(GL(2,4)) \times SL(2,4)$. So the central unit group $\mathrm{U}(Z(\mathbf{Z}SL(2,4)))$ of integral group ring $Z(\mathbf{Z}SL(2,4))$ of group $SL(2,4)(\cong A_5)$ is the subgroup of $\mathrm{U}(Z(\mathbf{Z}GL(2,4)))$. The central unit group $\mathrm{U}(Z(\mathbf{Z}A_5))$ can be found in [2].

Let β be a primitive 15th root of unity. The group $GL(2,4)$ has the character ξ of degree 3. The character field of ξ is $\mathbf{Q}(\beta+\beta^4)$. The local central unit $u_\xi(\lambda)$ can be determined for every nonzero $\lambda \in \mathbf{Q}(\beta+\beta^4)$ according to [3].

Theorem. *The central unit group* $\mathrm{U}(Z(\mathbf{Z}GL(2,4)))$ *is*

$$\langle -1\rangle \times Z(GL(2,4)) \times \mathrm{U}(Z(\mathbf{Z}SL(2,4))) \times \langle u_\xi((\beta+\beta^4)^{24})\rangle.$$

References

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[3] R. Zh. Aleev, Units of character fields and central units of integral group rings of finite groups. *Siberian Advances of Mathenatics.* **11** (2001) 1–33.