The Parallelization of Algorithms on The Base of The Conception of Q-determinant

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We describe the approach to parallelization algorithms based on their representation as Q-determinant. The proposed approach gives the possibility of the maximal parallelization of every algorithm if it enables the parallelization.

Let $\mathcal{A}$ be an algorithm to solve the algorithmic problem $\bar{y} = F(N, B)$ where N is a parameter dimension set of the problem, B is a set of input data, $\bar{y}$ is a set of output data. Let Q be a basic set of arithmetic and logical type operations. *The expression* is called the set of operands of arithmetic or logical type that use operations from Q. *Q-term* is the map from the problem dimension to a structured set of expressions that we need to calculate one of the output variables of the problem. The set of Q-terms can be unconditional, conditional and conditionally infinite according to the structure of expression set.

Q-determinant is the set of Q-terms that we need to calculate each of the problem output data [1]. Let an algorithm $\mathcal{A}$ be in the form of $y_i = f_i (i = 1, \ldots, m)$ where f_i is Q-term to calculate y_i, m is the number of output data. Then we consider that the algorithm $\mathcal{A}$ represents in the form of Q-determinant.

We consider the Gauss–Jordan solution of a system of linear equations as an example of representation of the algorithm in the form of Q-determinant. Let $A\bar{x} = \bar{b}$ be a system of linear equations, where $A = [a_{ij}]_{ij=1,\ldots,n}$ is a $n \times n$ invertible matrix, $\bar{x} = (x_1, \ldots, x_n)^T$, $\bar{b} = (a_{1,n+1}, \ldots, a_{n,n+1})^T$. At the first step we suppose that the leading element is the first nonzero element of the first row of the original matrix, and at k-th step $(2 \leq k \leq n)$ we select the first nonzero element of the k-th row that obtained at $(k-1)$-th step. Then the Q-determinant of Gauss–Jordan method consists of n conditional Q-terms and its representation in the form of Q-determinant has the shape

$$x_j = \left\{ (u_1, w_1^j), \ldots, (u_{n!}, w_{n!}^j) \right\} \ (j = 1, \ldots, n).$$

The realization of the algorithm in the form of Q-determinant is called the process of calculating the Q-terms $f_i (i = 1, \ldots, m)$ that are included in the Q-determinant. If the calculation of all Q-terms $f_i (i = 1, \ldots, m)$ is produced at the same time and as rapid as possible, i.e. the operations of the set are executed as soon as they are ready to perform, in this case we have *the most rapid implementation of the algorithm.*

If the algorithm has some representation as flowchart then it can be represent in the form of Q-determinant [2]. The software system QStudio [3] makes possible to calculate Q-determinant of any algorithm (if the algorithm has some representation as flowchart), to find the most rapid possible implementation and to build its execution plan for the parallel system.

References

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