Intersection of conjugated solvable subgroups in symmetric groups

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Assume that a finite group G acts on a set Ω. An element $x \in \Omega$ is called a *G-regular point* if $|xG| = |G|$, i.e. if the stabilizer of x is trivial. Define the action of the group G on Ω^k by the rule

$$g : (i_1, \ldots, i_k) \mapsto (i_1 g, \ldots, i_k g).$$

If G acts faithfully and transitively on Ω, then the minimal number k such that the set Ω^k contains a G-regular point is called the *base size* of G and is denoted by $b(G)$. For a positive integer m the number of G-regular orbits on Ω^m is denoted by $Reg(G, m)$ (this number equals 0 if $m < b(G)$). If H is a subgroup of G and G acts by the right multiplication on the set Ω of right cosets of H then G/H_G acts faithfully and transitively on the set Ω. (Here $H_G = \cap_{g \in G} H^g$.) In this case, we denote $b(G/H_G)$ and $Reg(G/H_G, m)$ by $b_H(G)$ and $Reg_H(G, m)$ respectively.

Thus $b_H(G)$ is the minimal number k such that there exist elements $x_1, \ldots, x_k \in G$ for which

$$H^{x_1} \cap \ldots \cap H^{x_k} = H_G.$$

Consider the problem 17.41 from "Kourovka notebook" [1]:

Let H be a solvable subgroup of finite group G and G does not contain nontrivial normal solvable subgroups. Are there always exist five subgroups conjugated with H such that their intersection is trivial?

The problem is reduced to the case when G is almost simple in [2]. Specifically, it is proved that if for each almost simple group G and solvable subgroup H of G condition $Reg_H(G, 5) \geq 5$ holds then for each finite nonsolvable group G and solvable subgroup H of G condition $Reg_H(G, 5) \geq 5$ holds.

We have proved the following theorem.

Theorem 1. *Let H be a solvable subgroup of an almost simple group G whose socle is isomorphic to A_n, $n \geq 5$. Then $Reg_H(G, 5) \geq 5$. In particular $b_H(G) \leq 5$.*

References

[1] Kourovka notebook; Edition 18, Novosibirsk 2014.

[2] E. P. Vdovin, On the base size of a transitive group with solvable point stabilizer. *Journal of Algebra and Application.* **11** (2012), N 1, 1250015 (14 pages)