Complication of the state orgraph for the queuing system with distinct channels

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Let us consider a queuing system T with distinct channels $P_1, ..., P_n, n > 1$ (distinct channels are the channels with heterogeneous service efficiencies $\mu_i \neq \mu_j$ and/or separate queues which lengths are $m_k \geq 0$, $1 \leq k \leq n$). The work of such a system presupposes the presence of dispatcher device D which distributes the arrived jobs between the channels in accordance with optimization criterion L. The known problem of "slow server"shows that if the channels are distinct according to their efficiency it is important to forward the arrived job to the most efficient channel P_i, even if a lower efficiency channel P_j, $\mu_i > \mu_j$ stands idle at that moment. Another case (when each channel has a separate queue) also presupposes that in a number of cases it is more profitable to forward a job to a queue of a more efficient channel (though its queue can be filled up to a greater extent), than to place it in a shorter line of a slow channel.

In this connection a simple representation of a queuing system T with a linear state orgraph $G(T)$: $S_0 \leftrightarrow S_1 \leftrightarrow S_2 \leftrightarrow ... \leftrightarrow S_N$ of a corresponding birth and death process is impossible. Here a state's index $j \in \overline{1, N}$ is equal to the number of jobs in a system, while its maximal value is equal to the sum of line's maximal lengths and a number of serving channels: $N = n + m_1 + ... + m_n$.

$G(T)$ state orgraph in case of distinct serving channels acquires a larger number of states and ramified nonlinear form [1], [2]. This is explained by the fact that there appear the distinct variants of states $S(k_1, ..., k_n)$ with the equal sum of jobs within a system: $0 \leq k_1 + ... + k_n \leq n + m_1 + ... + m_n$. Here $k_i \in \overline{0, m_i}$ is a sum of a number of jobs within channel i and within its queue, $i \in \overline{1, n}$. Dispatcher D should take into account the occupancy rate of the serving channels and their queues $S(k_1(t), ..., k_n(t))$ at the job arrival moment t. Moreover, let us interpret t as a time-step of the system operation. Let us describe the operation of the dispatcher D of the queuing system T with distinct channels by the finite state machine K which responds to the events of job arrival to the system's input or processed job release by one of the channels. Its input alphabet is $A = \{\alpha_0, \alpha_1, ..., \alpha_n\}$ where α_0 is a signal for job arrival to the queuing system T, α_i is a signal for the finish of job processing by the channel i, $i \in \overline{1, n}$.

Depending on the occupancy rates of the channels and their queues, dispatcher D should either reject the job arrived during the time-step t or forward it to the queue of one of the channels: $D(S(k_1(t), ..., k_n(t))) = S(k_1(t+1), ..., k_n(t+1)), k_i(t) \leq k_i(t+1), i \in \overline{1, n}$ Various optimality criteria L can be used in practice: reject or idle state probability minimization, minimization of the average time of the job standing in a queue or a system, maximization of the general capacity of the system and others. Each of these criteria is not equivalent to others which leads to various dispatching protocols $D = D(L)$. Let us suggest queuing system T operation simulation with a finite state machine $K(T, L)$ [1], [2] as a method to find the optimal dispatching protocol $D = D(L)$.

As a compensation for complication of $G(T)$ graph we get a possibility of universal representation by $K(T, L)$ finite state machine in case of non-Poisson arrival, i.e. when arrival is either non-ordinary or non-stationary. Moreover, finite state machines simulating queuing systems with additional conditions, such as jobs priority, sequential compilation procedure, return of unprocessed or partially processed jobs back to the system, etc., are built up uniformly.

References

[1] A.P. Kotenko., M.B. Bukarenko. Queuing system with distinct channels as a finite state machine [in Russian]. *SSTU Bulletin. Series: Phys. and Math.* **3(28)** (2012) 114-124.

[2] A.P. Kotenko., M.B. Bukarenko. Software set for simulation of a queuing system with heterogeneous servers and separate queues [in Russian]. *SSTU Bulletin. Series: Phys. and Math.* **2(31)** (2013) 50-57.