Algorithmic recognition by spectrum

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The set of element orders of a finite group G is called *the spectrum* and denoted by $\omega(G)$, and groups with the same spectrum are said to be *isospectral.* The following question seems to be natural: if $\mathcal{M}$ is a set of positive integers, does a group G with $\omega(G) = \mathcal{M}$ exist, and if so, can one describe all such groups? We are interested in algorithmic aspect of this problem under assumption that G is simple.

Given a finite group G, the set $\mathcal{M}$ is called *almost* G*-spectral*, if $\mathcal{M} \subseteq \omega(G)$, $\max \mathcal{M} = \max \omega(G)$, and $\omega(H) \neq \omega(\mathcal{M})$ for every simple group H whose spectrum differs from the spectrum of G. For a finite set $\mathcal{M}$, denote by $\Omega(\mathcal{M})$ the set of all simple groups G such that $\mathcal{M}$ is almost G-spectral.

We prove the following statement.

Theorem. *Let $\mathcal{M}$ be a finite set of positive integers, $m = |\mathcal{M}|$ and $M = \max \mathcal{M}$. Then, given $\mathcal{M}$, a group G such that G lies $\Omega(\mathcal{M})$ can be determined in time polynomial in $m \log M$.*

We also are going to discuss problems of effective generation of the spectrum of a group of Lie type.

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