Modules over group rings of locally finite groups with finiteness restrictions

Dashkova Olga
The Branch of Moscow state university in Sevastopol, Sevastopol, Russia

Let A be an $\mathbf{R}G$-module, $\mathbf{R}$ be an associative ring, G be a group. G is a finite-finitary group of automorphisms of A if $C_G(A) = 1$ and $A/C_A(g)$ is finite for any $g \in G$ [1]. Finite-finitary groups of automorphisms of A with additional restrictions were studied in [1].

Important finiteness conditions in group theory are the weak minimal condition on subgroups and the weak maximal condition on subgroups. Let G be a group, $\mathcal{M}$ be a set of subgroups of G. G is said to satisfy the weak minimal condition on $\mathcal{M}$-subgroups if for a descending series of subgroups $G_0 \geq G_1 \geq G_2 \geq \cdots \geq G_n \geq G_{n+1} \geq \cdots$, $G_n \in \mathcal{M}$, $n \in \mathbb{N}$, there exists the numder $m \in \mathbb{N}$ such that an index $|G_n : G_{n+1}|$ is finite for any $n \geq m$ [2]. Similarly G is said to satisfy the weak maximal condition on $\mathcal{M}$-subgroups if for an ascending series of subgroups $G_0 \leq G_1 \leq G_2 \leq \cdots \leq G_n \leq G_{n+1} \leq \cdots$, $G_n \in \mathcal{M}$, $n \in \mathbb{N}$, there exists the numder $m \in \mathbb{N}$ such that an index $|G_n : G_{n+1}|$ is finite for any $n \geq m$ [3]. These finiteness conditions were applied to investigate infinite dimensional linear periodic groups [4].

Let $\mathfrak{L}_{nf}(G)$ be the system of all subgroups H of G such that $A/C_A(H)$ is infinite. We say that G satisfies the condition W_{min-nf} if G satisfies the weak minimal condition on $\mathcal{M}$-subgroups where $\mathcal{M} = \mathfrak{L}_{nf}(G)$ and G satisfies the condition W_{max-nf} if G satisfies the weak maximal condition on $\mathcal{M}$-subgroups where $\mathcal{M} = \mathfrak{L}_{nf}(G)$.

Theorem 1. Let A be an $\mathbf{R}G$-module, $\mathbf{R}$ be an associative ring, G be a locally finite group. If G satisfies either W_{min-nf} or W_{max-nf} then either G is a Chernikov group or G is a finite-finitary group of automorphisms of A.

Let $G_{\mathfrak{S}}$ be the intersection of all normal subgroups K of G such that G/K is soluble.

Theorem 2. Let A be an $\mathbf{R}G$-module, $\mathbf{R}$ be an associative ring, G be a locally soluble periodic group. If G satisfies either W_{min-nf} or W_{max-nf} then $G/G_{\mathfrak{S}}$ is soluble.

References

[1] B. A. F. Wehrfritz, Finite-finitary groups of automorphisms. *J. Algebra Appl.* **1** (2002) 375–389.

[2] D. I. Zaitzev, Groups satisfying the the weak minimal condition. *Ukr. Math. J.* **20** (1968) 472-482.

[3] R. Baer, Polyminimaxgruppen. *Math. Ann.* **175** (1968) 1-43.

[4] J. M. Munoz-Escolano, J. Otal, N. N. Semko, Periodic linear groups with the weak chain conditions on subgroups of infinite central dimension. *Comm. Algebra.* **36** (2008) 749–763.