Decomposition of lattices of maximal antichains into the S-glued sum

Ilia Derendiaev, Alexander Popovich
Ural Federal University, Yekaterinburg, Russia

An antichain A of a poset P is called *maximal* if every element of P is comparable to an appropriate element of A. Consider the following relation on the set $MA(P)$ of all maximal antichains of P:

$$A \leqslant B \text{ iff for all } a \in A \text{ there exist } b \in B \text{ such that } a \leqslant b.$$

If P is finite, then $(MA(P), \leqslant)$ is a lattice. It is well-known (see [1]) that every finite lattice can be represented as the lattice of maximal antichains of a suitable poset. There are many such representations (see [2]), but all known representations of finite lattices as lattices of maximal antichains use posets of length 1.

In [3] V. Garg presented an application of lattices of maximal antichains in the theory of parallel computations. In his model elements of a poset were regarded as computations which are made by a single computer, and the formula $a > b$ means that the computation a starts after the computation b. Suppose that we want to minimize the number of computations without changing the lattice of maximal antichains. It is easy to show that the required poset is of maximal length. Then we obtain the following optimization problem:

Given a finite lattice L, find a finite poset P of maximal length such that $MA(P) \cong L$.

To solve this problem we use the notion of the S-glued sum (the definition can be founded in [4]). We prove the following theorem:

Let L be a finite lattice. Then the following statements are equivalent:
1) L is the S-glued sum for some finite lattice S of length k.
2) There exist a finite poset P of length k such that $AM(P) \cong L$.

In our talk we also discuss an algorithm that construct the corresponding poset.

References

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