Automorphisms of a distance-regular graph with intersection array $\{100, 66, 1; 1, 33, 100\}$

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A. A. Makhnev and D. V. Paduchikh have found [1] intersection arrays of distance-regular graphs, in which neighborhoods of vertices are strongly-regular graphs with second eigenvalue 3. A. A. Makhnev suggested the program to research of automorphisms of these distance-regular graphs. In this moment cases $\{100, 66, 1; 1, 33, 100\}$, $\{176, 150, 1; 1, 25, 176\}$ and $\{256, 204, 1; 1, 51, 256\}$ are not investigated.

In this paper are researching possible orders and subgraphs of fixed points of automorphisms of a hypothetical distance-regular graph with intersection array $\{100, 66, 1; 1, 33, 100\}$. Possible automorphisms of a strongly-regular graph with parameters (100,33,8,12) found in [2].

Theorem 1. *Let* Γ *be a distance-regular graph with intersection array* $\{100, 66, 1; 1, 33, 100\}$, $G = \mathrm{Aut}(\Gamma)$, *g be an element of G with prime order p and* $\Omega = \mathrm{Fix}(g)$ *contains along s vertices in t antipodal classes. Then* $\pi(G) \subseteq \{2, 3, 5, 7, 11, 29, 31, 101\}$ *and one of the following assertions holds:*

(1) Ω *is an empty graph and either* $p = 101$, $\alpha_1(g) = 101$, *or* $p = 3$, $\alpha_1(g) = 60m + 27l + 21$;

(2) $p = 31$, Ω *is a distance-regular graph with intersection array* $\{7, 4, 1; 1, 2, 7\}$;

(3) $p = 29$, Ω *is a distance-regular graph with intersection array* $\{13, 8, 1; 1, 4, 13\}$;

(4) $p = 11$ *and* $t = 2, 13, 24$;

(5) $p = 7$ *and either* Ω *is a distance-regular graph with intersection array* $\{16, 10, 1; 1, 5, 16\}$, *or* $t = 24, 31$;

(6) $p = 5$ *and* $t = 1, 16, 21, 26, 31$;

(7) $p = 3$, $s = 3$ *and* $t = 2, 5, ..., 32$;

(8) $p = 2$, *t is odd and either* $s = 3$, $t = 1, 3, 5, ..., 33$, *or* $s = 1$ *and* $t = 1, 3, 5, ..., 11$.

Theorem 2. *Let* Γ *be a distance-regular graph with intersection array* $\{100, 66, 1; 1, 33, 100\}$, *in which neighbourhoods of vertices are strongly-regular graphs with parameters* $(100, 33, 8, 12)$, $G = \mathrm{Aut}(\Gamma)$, *g be an element of G with prime order* $p > 2$ *and* $\Omega = \mathrm{Fix}(g)$ *is not empty graph, which contains along s vertices in t antipodal classes. Then* $\pi(G) \subseteq \{2, 3, 11, 101\}$ *and one of the following assertions holds:*

(1) $p = 11$, $s = 3$ *and* $t = 2$;

(2) $p = 3$, $s = 3$ *and either* $t = 5$, Ω *is an union of isolated* 5*-cliques, or* $t = 5, 8, ..., 17$ *and neighbourhoods of vertices in* Ω *are cocliques, or* $t = 11, 14, ..., 26$ *and neighbourhood of any vertex in* Ω *contains geodesic* 2*-path;*

(3) $p = 2$, *either* Ω *contained in antipodal class, or* $t = 5$ *and* Ω *is is an union of isolated* 5*-cliques and* $s = 1, 3$, *or neighbourhoods of vertices in* Ω *are unions of isolated cliques and* $s = 3$, $t = 3, 5$, *or neighbourhood of any vertex in* Ω *contains geodesic* 2*-path and* $s = 3$, $t = 7, 9, ..., 33$.

Corollary. *A distance-regular graph with intersection array* $\{100, 66, 1; 1, 33, 100\}$ *is not vertex-transitive.*

References

[1] A. A. Makhnev and D. V. Paduchikh, An exceptional strongly-regular graphs with eigenvalue 3 and their expansions. *Trudy of IMM UB RAS* **19** (2013) 167-174.

[2] M. S. Nirova, Edge-symmetric strongly-regular graphs with number of vertices is not greater 100, *Sibirean electr. Math. Reports* **10** (2013) 22-30.