

## Chain varieties of monoids

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Since the first half of 1960's, more than 200 articles appeared where the lattice of semigroup varieties is investigated. Many deep and interesting results were obtained here (see the survey articles [1, 2]). In contrast, only a few isolated facts is known so far about the lattice of monoid varieties. We know only two works devoted to examination of this lattice, namely [3, 4].

One of the first natural steps in investigation of varietal lattice of algebras of any type is a description of varieties whose lattice of subvarieties is a chain. Varieties with such a property are called *chain* varieties. Non-group chain varieties of semigroups and locally finite chain varieties of groups have been completely determined in [5] and [6] respectively. The problem of a complete description of chain varieties of groups seems to be extremely difficult. To confirm this claim, we refer to the fact that there exist uncountably many periodic group varieties whose subvariety lattice is the 3-element chain [7].

We completely classify all non-group chain varieties of monoids. The description is given in a language of identities and in terms of minimal forbidden subvarieties. We do not reproduce the description here because the corresponding list of varieties is quite lengthy. It consists of two countable series of varieties and 28 «sporadic» varieties. It is interesting to note that one of these two countable series of varieties appeared recently in the article [8] in connection with an investigation of so-called Cross varieties of monoids.

## Список литературы

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