Automorphisms of graph with intersection array $\{169, 126, 1; 1, 42, 169\}$

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We consider nondirected graphs without loops and multiple edges. For vertex a of a graph Γ the subgraph $\Omega_i(a) = \{b \mid d(a,b) = i\}$ is called i-neighboorhod of a in Γ. We set $[a] = \Gamma_1(a)$, $a^\perp = \{a\} \cup [a]$.

Degree of an vertex a of Γ is the number of vertices in $[a]$. Graph Γ is called regular of degree k, if the degree of any vertex is equal k. The graph Γ is called amply regular with parameters (v, k, λ, μ) if Γ is regular of degree k on v vertices, and $|[u] \cap [w]|$ is equal λ, if u adjacent to w, is equal μ, if $d(u,w) = 2$. Amply regular graph of diameter 2 is called strongly regular.

Jack Koolen suggested the problem investigation of distance-regular graphs whose local subgraphs are strongly regular graphs with the second eigenvalue at most t for some natural number t. For $t = 3$ A. Kagazezheva and A. Makhnev [1] proved the next result

Proposition. *Let Γ be a distance-regular graph with strongly regular local subgraphs having eigenvalue 3 and parameters $(v', k', 5, \mu')$. Then local subgraphs either isomorphic triangular graph $T(7)$ and Γ is a half graph of 7-cube, or have parameters $(169, 42, 5, 12)$ and Γ has intersection array $\{169, 126, 1; 1, 42, 169\}$.*

In this paper it is founded automorphisms of distance-regular graph with intersection array $\{169, 126, 1; 1, 42, 169\}$.

Theorem. *Let Γ be a distance-regular graph with intersection array $\{169, 126, 1; 1, 42, 169\}$, and local subgraphs of Γ are strongly regular with parameters $(169, 42, 5, 12)$, $G = \mathrm{Aut}(\Gamma)$, g — an element of G prime order $p > 2$ and $\Omega = \mathrm{Fix}(g)$ is nonempty graph containing s vertices in t antipodal classes. Then $\pi(G) \subseteq \{2, 3, 5, 7, 13, 17\}$ and one of the following holds:*

(1) *some $\langle g \rangle$-orbit on $\Gamma - \Omega$ contains geodesic 2-way, either $p = 7$ and $t = 2$, or $p = 5$ and Ω is a distance-regular graph with intersection array $\{9, 6, 1; 1, 2, 9\}$;*

(2) *some $\langle g \rangle$-orbit on $\Gamma - \Omega$ is clique, $p = 3$ and either $s = 4$, $t = 2, 5$ and Ω is the union of 4 isolated t-cliques, or $s = 1$ and Ω is 2-clique;*

(3) *every $\langle g \rangle$-orbit on $\Gamma - \Omega$ is coclique, either $p = 13$, Ω is an antipodal class, or $p = 5$ and $t = 40$, or $p = 3$, $s = 4$ and $t = 14$.*

Corollary. *Let Γ be a distance-regular graph with intersection array $\{169, 126, 1; 1, 42, 169\}$, and local subgraphs of Γ are strongly regular with parameters $(169, 42, 5, 12)$. If $G = \mathrm{Aut}(\Gamma)$ is nonsolvable group acting transitively on the vertex set of Γ, then $S = S(G)$ is an elementary abelian 2-group, $\bar{G} = G/S$ is isomorphic to $Sp_4(4)$, for any vertex $a \in \Gamma$ we have $G_a = 2^6 : (Z_3 \times A_5)$, S contains normal in G subgroup K of order 4, regular on each antipodal class, $|S : S_{\{F\}}| = 2$ fot antipodal class F, S/K is irreducible $F_2Sp_4(4)$-module of order $2^8, 2^{16}, 2^{32}$ and $C_S(f) = K$ for every element f of order 17 inG.*

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References

[1] A. Kagazegheva, A. Makhnev, On graphs with strongly regular local subgraphs having parameters (111,30,5,9) or (169,42,5,12), *Doklagy Akademii Nauk* 2014, V. 456, N 2. 135-139.