Automorphisms of local subgraphs of pseudogeometric graph for $pG_3(7,75)$

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We consider nondirected graphs without loops amd multiple edges. For vertex a of a graph Γ the subgraph $\Omega_i(a) = \{b \mid d(a,b) = i\}$ is called i-neighboorhod of a in Γ. We set $[a] = \Gamma_1(a)$, $a^{\perp} = \{a\} \cup [a]$. For a vertex subset S of a graph Γ we denote as $\Gamma(S)$ the set $\cap_{a\in S}([a] - S)$.

Degree of an vertex a of Γ is the number of vertices in $[a]$. Graph Γ is called regular of degree k, if the degree of any vertex is equal k. The graph Γ is called amply regular with parameters (v, k, λ, μ) if Γ is regular of degree k on v vertices, and $|[u] \cap [w]|$ is equal λ, if u adjacent to w, is equal μ, if $d(u,w) = 2$. Amply regular graph of diameter 2 is called strongly regular.

By $K_{m\times n}$ we denote the complete bipartite graph with m parties of order n. Graph on the set $X \times Y$ is called *$p \times q$-grid*, if $|X| = p$, $|Y| = q$, and pairs (x_1, y_1) and (x_2, y_2) are adjacent if and only if $x_1 = x_2$ or $y_1 = y_2$. By mK_n we denote the union of m isolated n-cliques.

A partial geometry $pG_\alpha(s,t)$ is a geometry of points and lines such that every line has exactly $s+1$ points, every point is on $t+1$ lines (with $s > 0,\ t > 0$) and for any antiflag (P, y) there are exactly α lines z_i containing P and intersecting y. In the case $\alpha = 1$ we have generalized quadrangle $GQ(s,t)$.

Point-graph of a geometry (P, L) of points and lines has P as a vertex set, and two vertices a, b are adjacent if a, b belong to some line. Point-graph of partial geometry $pG_\alpha(s,t)$ is strongly regular with parameters $v = (s+1)(1+st/\alpha)$, $k = s(t+1)$, $\lambda = (s-1)+(\alpha-1)t$, $\mu = \alpha(t+1)$. Strongly regular graph with this parameters for some natural numbers α, s, t is called pseudogeometric graph for $pG_\alpha(s,t)$.

A graph Γ is called t-izoregular, if for every $i \le t$ and for every i-vertex subset S the number $|\Gamma(S)|$ is depend only from isomorphic type of the subgraph induced by S. A graph on v vertices is called absolute izoregular, if it is $(v-1)$-izoregular. Finally t-izoregular graph Γ is called exactly t-izoregular, if it is not $(t+1)$-izoregular. Cameron [1] proved that every 5-izoregular graph Γ is absolute izoregular and is isomorphic pentagon, 3×3-grid, complete multipartite graph $K_{n\times m}$ or its complement. Further every exactly 4-izoregular graph is pseudogeometric for $pG_r(2r, 2r^3 + 3r^2 - 1)$ or its complement. Let $Izo(r)$ be a pseudogeometric graph for $pG_r(2r, 2r^3 + 3r^2 - 1)$. For $r = 1$ we have the point graph of $GQ(2,4)$, and for $r = 2$ we have MacLaughlin graph.

For every vertex a of a graph $Izo(r)$ the subgraph $\Gamma(a)$ is pseudogeometric for $pG_{r-1}(2r-1, r^3+r^2-r-1)$. Makhnev [2] proved that pseudogeometric graph for $pG_{r-1}(2r-1, r^3+r^2-r-1)$ does not exist for $r = 3$. Automorphisms of 2-neighboorhod Σ of some vertex of $Izo(3)$ and local subgraphs of Σ were determined by M. Nirova, M. Isakova and A. Tokbaeva [3], [4], [5].

Graph $Izo(4)$ has parameters $(3159, 1408, 532, 704)$ and for any vertex a subgraph $\Sigma = [a]$ is pseudogeometric for $pG_3(7,75)$ and has parameters $(1408, 532, 156, 228)$. Further, for any vertex $b \in \Sigma$ subgraph $\Delta = \Sigma(b)$ is pseudogeometric for $pG_2(6,25)$ and has parameters $(532, 156, 30, 52)$, subgraph $\Delta' = \Sigma_2(b)$ is strongly regular with parameters $(875, 304, 78, 120)$. In this paper automorphisms of strongly regular with parameters $(532, 156, 30, 52)$ are determined.

Theorem. *Let Γ be a strongly regular with parameters $(532, 156, 30, 52)$, $G = \mathrm{Aut}(\Gamma)$, g is an element of prime order p of G and $\Omega = \mathrm{Fix}(g)$. Then $\pi(G) \subseteq \{2,3,5,7,11,13,17,19,23,29\}$ and one of the following holds:*

(1) *Ω is empty graph, either $p = 19$ and$\alpha_1(g) = 152$, or $p = 7$ and $\alpha_1(g) = 210l - 28$, or $p = 2$ and $\alpha_1(g) = 30l + 16$;*

(2) *Ω is n-cliqwue, either $p = 3$, $n = 1$ and $\alpha_1(g) = 90l + 36$, or $p = 5$, $n = 2$ and $\alpha_1(g) = 150l - 20$ or $n = 7$ and $\alpha_1(g) = 150l - 30$;*

(3) *Ω is l-coclique, either $p = 2$ and $\alpha_1(g) = 4m + 152 - 60l$, or $p = 13$ and $\alpha_1(g) = 13(4s - 30t - 14)$, where $m = 13s - 1$;*

(4) *Ω contains geodesic 2-way and $p \le 29$.*

This work was supported by the grant of Russian Science Foundation, project no. 15-11-10025.

References

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