Spectral properties of the Star graph

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Spectral properties of Cayley graphs on the symmetric group Sym_n generated by transpositions have studied intensively last years. In 2000 it was shown by J. Friedman [1] that the Cayley graph on Sym_n with respect to a set of $n-1$ transpositions has the smallest non-zero eigenvalue $\lambda_2 \leqslant 1$, with equality iff for some i we have $T = \{(i,j)|j \neq i\}$. The multiplicity of this eigenvalue is

$$mul(\lambda_2) \geqslant n-1. \tag{1}$$

For example, if $T = \{(1,2),(2,3),\ldots,(n-1,n)\}$ then we have the Bubble-sort graph whose spectral properties were investigated by R. Bacher in [2].

In this paper we study spectral properties of the *Star graph* S_n that is the Cayley graph on Sym_n with the generating set $T = \{(1,2),(1,3),\ldots,(1,n)\}$. In 2009 A. Abdollahi and E. Vatandoost conjectured [3] that the spectrum of S_n is integral, moreover it contains all integers in the range from $-(n-1)$ up to $n-1$ (with the sole exception that when $n \leqslant 3$, zero is not an eigenvalue of S_n). This conjecture was proved by R. Krakovski and B. Mohar [4] in 2012.

We investigate multiplicity of eigenvalues of the Star graph S_n. Using the standard representation theory [5] their exact values were found for $4 \leqslant n \leqslant 13$. The obtained data show an oscillating distribution of eigenvalue multiplicities. One can assume that this behavior of multiplicities will be also kept for large n. Let us note that typically the distribution of eigenvalue multiplicities for known distance-regular graphs is unimodal. However, the Star graph is not distance-regular. It is also shown that the low bound (1) for $mul(\lambda_2)$ is achieved only for $2 \leqslant n \leqslant 5$ in S_n. The following result is given.

Theorem. *The values* $\pm(n-2)$ *are eigenvalues of* S_n *with multiplicity* $(n-2)(n-1)$.

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