On adjacency for the prime graph of a finite simple group

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The prime (or Gruenberg-Kegel) graph $\Gamma(G)$ of a finite group G is an undirected simple graph whose vertex set is the set $\pi(G)$ of all prime divisors of $|G|$ and two vertices p and q are adjacent if and only if there exists an element of order pq in G. If $|G|$ is even then we denote by $\pi_1(G)$ the connected component of $\Gamma(G)$ containing 2. It is very known (see, for example, [1, 2]) that the prime graph of any finite simple non-abelian group is not complete. We prove the following theorem which strengthen this result.

Theorem. *Let G be a finite simple nonabelian group. Then there exist in the graph $\Gamma(G)$ two nonadjacent odd vertices which do not divide $|Out(G)|$, moreover it is possible to take such vertices in $\pi_1(G)$, except when G is isomorphic to one of the following groups: M_{11}, M_{22}, J_1, J_2, J_3, HiS, A_n ($n \in \{5, 6, 7, 9, 12, 13\}$), $A_1(q)$ $(q > 3)$, $A_2^{\varepsilon}(q)$ ($q = p^m > 2$, p is a prime, $m \in \mathbb{N}$, $\varepsilon \in \{+, -\}$ and either $\pi(q + \varepsilon 1) = \{2\}$ or p divides $2m$), ${}^2A_3(3)$, ${}^2A_5(2)$, $C_3(2)$, $C_2(q)$ $(q > 2)$, $D_4(2)$, ${}^2B_2(q)$ $(q = 2^{2k+1} > 2)$, $G_2(q)$ $(q = 3^k)$.*

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