The automorphism group of finite semifield

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A *semifield* is an algebraic structure $\langle W, +, \circ \rangle$, satisfying the following axioms:
1) $\langle W, + \rangle$ is abelian group;
2) $\langle W^*, \circ \rangle$ is a loop;
3) $x \circ (y+z) = x \circ y + x \circ z$ and $(y+z) \circ x = y \circ x + z \circ x$ for all $x, y, z \in W$.

The projective plane π coordinatizing by semifield W is called a *semifield plane.* Let π be a semifield plane of order p^n, p be prime. We can represent the coordinatizing semifield of such a plane as a n-dimensional linear space over $\mathbb{Z}_p$, with multiplication law

$$x \circ y = x\theta(y), \quad x, y \in W.$$

Here $\theta : W \to GL_n(p) \cup \{0\}$ is a bijective mapping, satisfiyng the conditions:
1) $\theta(y+z) = \theta(y) + \theta(z)$ $\forall y, z \in W$;
2) $\theta(0, 0, \ldots, 0, 0) = 0$, $\theta(0, 0, \ldots, 0, 1) = E$ (identity matrix).

We shall call the matrix set $R = \{\theta(y) | y \in W\}$ a *regular set.*

Theorem. *The bijective mapping $x \to xA$, $x \in W$, is an automorphism of semifield W for $A \in GL_n(p)$ if and only if*

$$A^{-1}\theta(y)A = \theta(yA) \qquad \forall y \in W.$$

Moreover, the matrix

$$\begin{pmatrix} A & 0 \\ 0 & A \end{pmatrix}$$

determines the collineation of semifield plane π, that fixes a triangle $(0, 0)$, (0), (∞) and a line $y = x$.

We used the matrix representation of automorphism to construct the autotopism subgroup of semifield plane and automorphism group of coordinatizing semifield of some small orders, odd and even. Also the matrix representation of inner automorphisms [1] of finite semifield is determined.

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References

[1] G. P. Wene, Inner automorphisms of finite semifields. *Note Mat.* **29** (2009) 231-242.