# An injective map from the set of maximum independent sets in a Doob graph to the set of 4-ary distance-2 MDS codes

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The Cartesian product $D(m,n) \stackrel{\mathrm{def}}{=} \mathrm{Sh}^m \times K_4^n$ of $m$ copies of the Shrikhande graph Sh (see the left part of Fig. 1) and $n$ copies of the complete graph $K_q$ of order $q = 4$ is called a Doob graph if $m > 0$, while $D(0,n)$ is the Hamming graph $H(n,4)$ (in general $H(n,q) \stackrel{\mathrm{def}}{=} K_q^n$). The Doob graph $D(m,n)$ is a distance-regular graph with the same parameters as $H(2m+n,4)$. It is easy to see that the independence number of this graph is $4^{2m+n-1}$. The maximum independent sets in the Hamming graphs are known as the distance-2 MDS codes, or the Latin hypercubes (in the last case, one coordinate is usually considered as a function of the other coordinates). It is natural to generalize these notions to the maximum independent sets in Doob graphs; however, for generalized Latin hypercubes in $D(m,n)$, we need at least one $K_4$ coordinate, i.e., $n > 0$. There are 4 trivial MDS codes in $D(0,1)$; 24 equivalent distance-2 MDS codes in $D(0,2)$ (16 of them can be found in Fig. 1); 16 distance-2 MDS codes in $D(1,0)$ (see Fig. 1), which form two equivalence classes.

The goal of the current correspondence is to describe a rather simple recursive way to map injectively the set $\mathrm{MDS}_{m,n}$ of distance-2 MDS codes in $D(m,n)$ into $\mathrm{MDS}_{0,2m+n}$. At first, we define the map $\kappa$ from $\mathrm{MDS}_{1,0}$ into $\mathrm{MDS}_{0,2}$, see Fig. 1. This map has the following important property: two MDS codes $M'$ and $M''$ in $D(1,0)$ intersect if and only if their images $\kappa M'$ and $\kappa M''$ intersect. It follows that $\kappa$:

$$\kappa M \stackrel{\mathrm{def}}{=} \big\{(x_1,...,x_m,z_1,z_2,y_1,...,y_n) \in D(m,n+2) \,\big|\, (z_1,z_2) \in \kappa\{v \in \mathrm{Sh} \mid (x_1,...,x_m,v,y_1,...,y_n) \in M\}\big\}$$

maps $\mathrm{MDS}_{m+1,n}$ into $\mathrm{MDS}_{m,n+2}$. Then, $\kappa^m$ maps $\mathrm{MDS}_{m,n}$ into $\mathrm{MDS}_{0,2m+n}$. A constructive characterization of the class $\mathrm{MDS}_{0,2m+n}$ can be found in [1]; using the map $\kappa^m$, it is possible to extract some information on $\mathrm{MDS}_{m,n}$ for arbitrary $m$. In particular, $|\mathrm{MDS}_{m,n}| = 2^{2^{2m+n}(1+o(1))}$ (by comparison, the number of all vertex subsets in $D(m,n)$ is $2^{2^{4m+2n}}$).

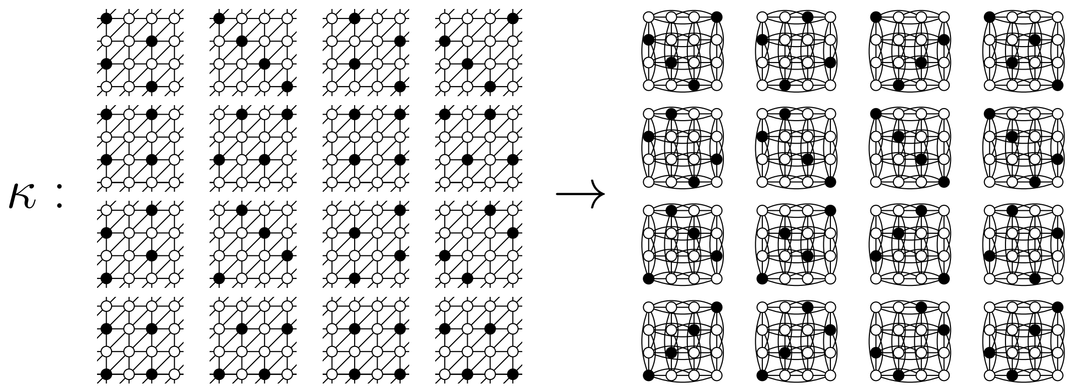

Figure 1: The 16 maximum independent sets in Sh and the corresponding independent sets in $K_4^2$

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## References

[1] D. S. Krotov, V. N. Potapov, $n$-Ary quasigroups of order 4 // SIAM J. Discrete Math. 2009. Vol. 23, no. 2. P. 561–570.