Compositional structure of groups isospectral go $U_3(3)$

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In this work only finite groups are studied. The *spectrum* $\omega(G)$ of a group G is the set of its element orders. By a *section* of G we mean a quotient group H/N, where $N, H \leq G$ and $N \trianglelefteq H$. Groups G and H are called *isospectral*, if $\omega(G) = \omega(H)$. Let ω be a subset of natural numbers. Following [1], we call a group G *critical with respect to* ω (or ω*-critical*), if ω coincides with the spectrum of G and does not coincide with the spectrum of any proper section of G.

If a simple group L has infinitely many groups isospectral to L, then it is important to study critical groups isospectral to L. In [2, 3] the complete description is given of critical groups isospectral to non-abelian simple alternating and sporadic groups and also the special linear group $SL_3(3)$.

In this work we study groups critical with respect to the spectrum of the projective special unitary group $U_3(3)$. In particular, we prove the following

Theorem. *Let G be a group isospectral to $U_3(3)$ that contains a normal subgroup N, such that $G/N \simeq PGL_2(7)$. Then N is a 2-group and every G-chief factor of N is isomorphic to a 6-dimensional module of the group $PGL_2(7)$. Also $G = NH$ for some subgroup $H \simeq PGL_2(7)$. If in addition G is critical with respect to $\omega(U_3(3))$, then $|N| = 2^6$.*

Moreover, H has a representation $\langle a, b, c \mid a^2 = b^3 = c^2 = (ab)^7 = (ac)^2 = (bc)^2 = [a, b]^4 = 1\rangle$ and if we regard N as a vector space over $GF(2)$ then a base of N can be chosen in such a way that the action of H on N is defined by the following matrices:

$$a \sim \begin{pmatrix} 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot & \cdot & \cdot \\ 1 & \cdot & 1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot & 1 \end{pmatrix}, \ b \sim \begin{pmatrix} \cdot & 1 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot & \cdot & \cdot \\ 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & 1 \\ \cdot & \cdot & \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 & \cdot \end{pmatrix}, \ c \sim \begin{pmatrix} \cdot & \cdot & \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & 1 \\ 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot & \cdot & \cdot \end{pmatrix}.$$

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References

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[2] *Lytkin Y. V.* On groups critical with respect to a set of natural numbers // Siberian Electronic Mathematical Reports. 2013. V. 10. P. 666–675; http://semr.math.nsc.ru/

[3] *Lytkin Y. V.* Groups critical with respect to the spectra of alternating and sporadic groups // Siberian Mathematical Journal. 2015. V. 56, N 1. P. 101-106.