Automorphisms of distance-regular graph with intersection array $\{204, 175, 48, 1; 1, 12, 175, 204\}$

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We consider nondirected graphs without loops amd multiple edges. For vertex a of a graph Γ the subgraph $\Omega_i(a) = \{b \mid d(a, b) = i\}$ is called i-neighboorhod of a in Γ. We set $[a] = \Gamma_1(a)$.

Degree of an vertex a of Γ is the number of vertices in $[a]$. Graph Γ is called regular of degree k, if the degree of any vertex is equal k. The graph Γ is called amply regular with parameters (v, k, λ, μ) if Γ is regular of degree k on v vertices, and $|[u] \cap [w]|$ is equal λ, if u adjacent to w, is equal μ, if $d(u, w) = 2$. Amply regular graph of diameter 2 is called strongly regular.

Distance-regular graph Γ with intersection array $\{204, 175, 48, 1; 1, 12, 175, 204\}$ is $AT4(4, 6, 5)$-graph [1]. Antipodal quotient $\bar{\Gamma}$ has parameters $(800, 204, 28, 60)$.

In this paper automorphisms of distance-regular graph Γ with intersection array $\{204, 175, 48, 1; 1, 12, 175, 204\}$ and of antipodal quotient $\bar{\Gamma}$ are investigated.

Theorem 1. *Let Γ be a strongly regular with parameters $(800, 204, 28, 60)$, $G = \mathrm{Aut}(\Gamma)$, g be an element of prime order p of G and $\Omega = \mathrm{Fix}(g)$. Then $\pi(G) \subseteq \{2, 3, 5, 7, 17\}$ and one of the following holds:*

(1) *Ω is empty graph, either $p = 5$, $\alpha_1(g) = 200l$, or $p = 2$ and $\alpha_1(g) = 40m$;*

(2) *Ω is n-cliqwue, either $p = 17$, $n = 1$ and $\alpha_1(g) = 204$, or $p = 5$, $n = 5$ and $\alpha_1(g) = 200s + 20$ or $p = 7$, $n = 2$ and $\alpha_1(g) = 280t + 168$;*

(3) *Ω is l-coclique, either $p = 3$, $l = 3m + 2$ and $\alpha_1(g) = 120t + 12m + 48$, or $p = 2$, $\alpha_1(g) = 80t + 4l$, where $l = 8, 10, ..., 92$;*

(4) *Ω is the union of m isolated 5-cliques, $2 \le m \le 5$, $\alpha_1(g) = 200s + 20m$;*

(5) *Ω contains geodesic 2-way and either*

(*i*) *$p = 3$, Ω is the union of $3m + 1$ isolated subgraphs $K_{4\times2}$ and $\alpha_1(g) = 96m + 120t + 72$, or*

(*ii*) *$p = 2$, $|\Omega| = 2l \le 240$, $\lambda_\Omega = 0, 2, ..., 26$, degrees of vertices in Ω equal $0, 2, ..., 34$ and $\alpha_1(g) = 80t + 8l$.*

Theorem 2. *Let Γ be a distance-regular graph Γ with intersection array $\{204, 175, 48, 1; 1, 12, 175, 204\}$, $G = \mathrm{Aut}(\Gamma)$, g be an element of prime order p of G and $\Omega = \mathrm{Fix}(g)$ contains s vertices in t antipodal classes. Then $\pi(G) \subseteq \{2, 5, 7, 17\}$ and one of the following holds:*

(1) *Ω is empty graph, either $p = 5$, $\alpha_1(g) = 200(4+m-l)$, $\alpha_2(g) = 1000l$ and $\alpha_3(g) = 200(16-m-4l)$, or $p = 2$, $\alpha_1(g) = 80(4 + m - l)$, $\alpha_2(g) = 400l$ and $\alpha_3(g) = 80(46 - m - 4l)$;*

(2) *g induces trivial automorphism of antipodal quotient $\bar{\Gamma}$, $p = 5$ and $\alpha_4(g) = v$;*

(3) *Ω is the antipodal class of Γ, $p = 17$, $\alpha_1(g) = 340 + 680n$, $\alpha_2(g) = 2975$ and $\alpha_3(g) = 680(1 - n)$;*

(4) *Ω is the union of two antipodal classes, $p = 7$, $\alpha_1(g) = 910 + 280n - 70l$, $\alpha_2(g) = 350l$, $\alpha_3(g) = 3080 - 280l - 280n$, $l = 1, 5, 9$;*

(5) *$p = 5$, $t = 5, s = 5$, $\alpha_1(g) = 700 + 200(m - l)$, $\alpha_2(g) = 1000l - 125$ and $\alpha_3(g) = 200(17 - m - 4l)$.*

Corollary. *Let Γ be a distance-regular graph Γ with intersection array $\{204, 175, 48, 1; 1, 12, 175, 204\}$. Then group $G = \mathrm{Aut}(\Gamma)$ is solvable.*

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References

[1] A. Makhnev, D. Paduchikh, On strongly regular graph with eigenvalue μ and its extensions, Proceedings of the Steklov Institute of Mathematics, June 2014, Volume 285, Issue 1 Supplement, S128-S135.