Strongly regular graphs with strongly regular local subgraphs having second eigenvalue 5

Alexander Makhnev, Dmitrii Paduchikh
N.N. Krasovskii Institute of Mathematics and Mechanics UB RAS, Yekaterinburg, Russia

We consider nondirected graphs without loops amd multiple edges. For vertex a of a graph Γ the subgraph $\Omega_i(a) = \{b \mid d(a,b) = i\}$ is called i-neighboorhod of a in Γ. We set $[a] = \Gamma_1(a)$, $a^\perp = \{a\} \cup [a]$.

Degree of an vertex a of Γ is the number of vertices in $[a]$. Graph Γ is called regular of degree k, if the degree of any vertex is equal k. The graph Γ is called amply regular with parameters (v,k,λ,μ) if Γ is regular of degree k on v vertices, and $|[u]\cap[w]|$ is equal λ, if u adjacent to w, is equal μ, if $d(u,w) = 2$. Amply regular graph of diameter 2 is called strongly regular.

A partial geometry $pG_\alpha(s,t)$ is a geometry of points and lines such that every line has exactly $s+1$ points, every point is on $t+1$ lines (with $s>0,\ t>0$) and for any antiflag (P,y) there are exactly α lines z_i containing P and intersecting y. In the case $\alpha = 1$ we have generalized quadrangle $GQ(s,t)$.

Jack Koolen suggested the problem investigation of distance-regular graphs whose local subgraphs are strongly regular graphs with the second eigenvalue at most t for some natural number t. Recently this problem was solved for $t = 3$. At present near finishing the case $t = 4$. We begin the investigation of the case $t = 5$. In [1] was obtained the reduction to the exceptional local subgraphs. Let *Gamma* be a distance regular graph of diameter $d \geq 3$. Then $c_2 \leq b_1$. A. Makhnev and D. Paduchikh found parameters of exceptional strongly regular graphs with the second eigenvalue 5, which may be local subgraphs in amply regular graphs with $\mu \leq b_1$.

In this paper it is determined parameters of strongly regular graphs with strongly regular local subgraphs havung the second eigenvalue 5.

Theorem. *Let Γ be a strongly regular graph with strongly regular local subgraphs havung the second eigenvalue* 5. *Then Γ has parameters* $(176,49,12,14)$, $(209,100,45,50)$, $(259,42,5,7)$, $(356,85,30,17)$, $(806,625,480,500)$, $(1464,1225,1020,1050)$ *or local subgraphs are exceptional and Γ has parameters*

(1) $(100,36,14,12)$, $(100,77,60,56)$, $(189,100,55,50)$, $(169,112,75,72)$, $(330,105,40,30)$, $(345,120,35,45)$, $(400,210,110,110)$, $(512,133,24,38)$, $(550,225,80,100)$, $(560,325,180,200)$, $(605,280,117,140)$, $(680,175,30,50)$, $(846,260,70,84)$, $(946,273,80,78)$, $(990,345,120,120)$,

(2) $(1003,300,65,100)$, $(1016,259,42,74)$, $(1036,375,110,150)$, $(1080,260,70,60)$, $(1090,441,152,196)$, $(1122,209,16,44)$, $(1199,550,225,275)$, $(1200,605,280,330)$, $(1458,329,40,84)$, $(1520,385,60,110)$, $(1577,400,105,100)$, $(1976,175,30,14)$;

(3) $(2025,680,175,255)$, $(2032,1275,770,850)$, $(2034,437,100,92)$, $(2209,624,161,182)$, $(2420,885,260,360)$, $(2508,1199,550,594)$, $(2809,540,77,110)$, $(3250,1305,440,580)$, $(3481,960,245,272)$, $(3844,630,68,110)$, $(3872,343,54,28)$, $(3888,1625,580,750)$, $(3950,385,60,35)$;

(4) $(4256,259,42,14)$, $(4418,637,96,91)$, $(4496,1015,150,252)$, $(4512,650,55,100)$, $(4706,3625,2760,2900)$, $(4941,1520,385,504)$, $(5074,969,176,187)$, $(5625,1520,385,420)$, $(5820,2783,1270,1386)$, $(7139,3250,1305,1625)$, $(7280,1015,150,140)$, $(9801,1600,205,272)$.

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References

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