Strongly regular graphs with nonprincipal eigenvalue 5 and its extensions

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We consider nondirected graphs without loops amd multiple edges. For vertex a of a graph Γ the subgraph $\Omega_i(a) = \{b \mid d(a,b) = i\}$ is called i-neighboorhod of a in Γ. We set $[a] = \Gamma_1(a)$, $a^{\perp} = \{a\} \cup [a]$.

Degree of an vertex a of Γ is the number of vertices in $[a]$. Graph Γ is called regular of degree k, if the degree of any vertex is equal k. The graph Γ is called amply regular with parameters (v, k, λ, μ) if Γ is regular of degree k on v vertices, and $|[u] \cap [w]|$ is equal λ, if u adjacent to w, is equal μ, if $d(u,w) = 2$. Amply regular graph of diameter 2 is called strongly regular.

A partial geometry $pG_{\alpha}(s,t)$ is a geometry of points and lines such that every line has exactly $s+1$ points, every point is on $t+1$ lines (with $s > 0,\ t > 0$) and for any antiflag (P, y) there are exactly α lines z_i containing P and intersecting y. In the case $\alpha = 1$ we have generalized quadrangle $GQ(s,t)$.

Jack Koolen suggested the problem investigation of distance-regular graphs whose local subgraphs are strongly regular graphs with the second eigenvalue at most t for some natural number t. In [1] the solving of Koolen problem in the case $t = 3$ was began.

We begin the investigation of the case $t = 5$.

Strongly regular graph Γ with the second eigenvalue $m-1$ is called exceptional if Γ does not belong the following list:

(1) the union of isilated m-cliques;

(2) pseudogepmetric graph for $pG_t(t+m-1,t)$;

(3) the complement of pseudogepmetric graph for $pG_m(s,m-1)$;

(4) conference graph with parameters $(4\mu+1, 2\mu, \mu-1, \mu)$, $\sqrt{4\mu+1} = m-1$.

In this paper it is obtained reduction to locally exceptional graphs.

Theorem. *Let Γ be a distance-regular graph with strongly regular local subgraphs havung the second eigenvalue t, $4 < t \leq 5$, u is a vertex of Γ. Then $[u]$ is an exceptional strongly regular graph, or one of the following holds:*

(1) *$[u]$ is the union of isilated 6-cliques;*

(2) *$[u]$ is the pseudogepmetric graph for $pG_{s-5}(s,s-5)$ and either*

(i) Γ is strongly regular graph with parameters $(176,49,12,14)$, $(209,100,45,50)$, $(806,625,480,500)$, $(1464,1225,1020,1050)$, and $s = 6,9,24,34$ respectively, or

(ii) $s = 6$ and Γ is Johnson graph $J(14,7)$, or its standard quotient or graph with intersection array $\{49,36,1;1,12,49\}$, or

(iii) $s = 7$ and Γ has intersection array $\{64,42,1;1,21,64\}$, or

(iv) $s = 10$ and Γ is Taylor graph;

(3) *$[u]$ the complement of pseudogepmetric graph for $pG_6(s,5)$, Γ is strongly regular graph with parameters $(259,42,5,7)$, $(356,85,30,17)$, and $s = 8,6$ respectively, or $s = 12$ and Γ is Taylor graph;*

(4) *$[u]$ is the conference graph with parameters $(4l+1, 2l, l-1, l)$, $l \in \{21,22,24,25,27,28,29,30\}$ and Γ is Taylor graph.*

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References

[1] A. Makhnev, Strongly regular graphs with nonprincipal eigenvalue 4 and its extensions. *Izvestiya of Gomel University*, 2014. V. 84, N 3. 84-85.