# Invariants of virtual links

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Virtual knot theory has been introduced by Kauffman [1] as a generalization of the classical knot theory. Virtual knots (and links) are represented as generic immersions of circles in the plane (virtual link diagrams) where double points can be classical (with the usual information on overpasses and underpasses) or virtual. Virtual link diagrams are equivalent under ambient isotopy and some types of local moves (generalized Reidemeister moves).

Using virtual generalized Reidemeister moves we can introduce a notion of "virtual"braids. Virtual braids on $n$ strands form a group denoted by $VB_n$. The relation between virtual braids and virtual knots (and links) are completely determined by a generalization of Alexander and Markov Theorem [2, 3]. It is worth to mention that for virtual braids an Alexander-like theorem states that any virtual link can be represented as the closure of a virtual braid.

In the classical case it is known that the braid group embeds into $\mathrm{Aut}(F_n)$ by Artin representation which is a local one. Wada [6] classified all local representations of the braid group $B_n$ into $\mathrm{Aut}(F_n)$. There are four types. It is proved [7] that these representations are faithful.

**Proposition** *For every Wada representation*

$$w_1^r, w_2, w_3, w_4 : B_n \to AutF_n,\ r \in \mathbb{Z}$$

*it is possible to construct the corresponding representation of the virtual braid group*

$$W_1^r, W_2, W_3, W_4 : VB_n \to AutF_{n+1},\ r \in \mathbb{Z},$$

*such that the restriction each of them onto $B_n$ is coincide with the corresponding Wada representation, i.e.*

$$W_k|_{B_n} = w_k,\ k = 1, 2, 3, 4.$$

Analogously to the way shown in [4] we introduce the notion of the group of the virtual link $G(vL)$ for representations of the virtual braid group $W_k,\ k = 1, \dots, 4$. Let $vL = \widehat{\beta_v}$ be a closure of the virtual braid, where $\widehat{\beta_v} \in VB_n$. We define

$$G_k(vL) = \langle x_1, x_2, \dots, x_n, y \parallel x_i = W_k(\beta_v)(x_i),\ i = 1, 2, \dots, n\rangle,\ k = 1, \dots, 4.$$

**Theorem.** *The constructed groups $G_k(vL),\ k = 1, \dots 4$, are invariants of the virtual link $vL$.*

## References

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