On a generalization of relations schemas, related to groups $U_3(q)$ and ${}^2G_2(3^{2l+1})$

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Let X be a finite set, R_i $(i = 0, 1, \dots, d)$ – binary relations on X, that satisfy conditions of *symmetric commutative associative scheme of relations on d classes* (see definitions in [1]), except condition of constancy number of intersections p^1_{ij}. Pair $(X, \{R_i\}_{\{0\leqslant i\leqslant d\}}) = \mathcal{X}(X)$ is called *scheme*, $\Gamma^{(i)} = (X, R_i)$ – *graph of i-th relation*. Scheme $\mathcal{X}(X)$ is called *scheme of cliques* if:

1. Graph $\Gamma^{(1)}$ of 1st relation is disconnected with $n + 1$ connected components – cliques K_1, K_2, ..., K_{n+1} with $d - 1$ vertices each.

2. If x is fixed vertex of clique K_s, $K_t \neq K_s$, y iterates over vertices from K_t, then in $(x, y) \in R_i$ index i iterates over indices from $2, 3, \dots, d$.

If scheme of cliques satisfies the additional condition

3. If i, j, k and s, t, l are arbitrary sets of pairwise distinct indices from $\{2, 3, \dots, d\}$, then $p^k_{ij} = p^l_{st}$, $p^k_{ii} = p^l_{ss}$, $p^i_{ii} = p^s_{ss}$, and $p^1_{ij} \in \{0, r\}$, $r \neq 0$,

then it called scheme of cliques with the *absolute number of intersections*.

In [2] was announced existence of these schemes on the class X of conjugate elements of prime order p from centers of p-Sylow subgroups in group $G \in \{L_2(q), Sz(q), U_3(q)\}$ (with even $q \geqslant 4$ in last case) with $q = p^m$, where p is prime. Also was announced distance-regularity with array of intersections $\{n, n - p^i_{ii} - 1, 1; 1, p^k_{ii}, n\}$ of their graphs $\Gamma^{(i)}$ i-th relations with $i \in \{2, 3, \dots, d\}$, $k \neq i$, where $p^k_{ii} = p^i_{ii} = (n - 1)/(d - 1)$. Proved

Theorem. *If $G \in \{U_3(q), {}^2G_2(q)\}$, $q = p^m$ – degree of odd prime number p with $q = 3^{2l+1}$ in case $G = {}^2G_2(q)$, X – class of conjugate elements of order p from centers of p-Sylow subgroup of group G, then on X can be defined relations R_i $(i = 0, 1, \dots, d)$, such that $(X, \{R_i\}_{\{0\leqslant i\leqslant d\}})$ – scheme of cliques with $d = q$, $n = q^3$ и $p^k_{ij} = p^k_{ii} = p^i_{ii} = 2(q^2 + q + 1)$ for all $i, j, k \in \{2, 3, \dots, d\}$.*

References

[1] E. Bannai, T. Ito, Algebraic Combinatorics I: Association Schemees, Benjamin-Cummings Lecture Note Ser.58, The Benjamin/Cummings Publishing Company, Ins., London (1984)

[2] I. T. Mukhamet'yanov, On a generalization of association schemes and related antipodal distance regular graphs with diameter 3 // Maltsev Meeting: Abstracts of Intern. Conf. Novosibirsk, 2014. P. 73 (in Russian).