

## Generalized supersoluble finite groups and mutually permutable products

Evgeniy Myslovets

*F. Scorina Gomel State University, Gomel, Belarus*

Only finite groups are considered. In the paper [1] V.A. Vedernikov introduced the notion of  $c$ -supersoluble group. Recall that a group  $G$  is called  $c$ -supersoluble if  $G$  has a chief series whose chief factors are simple. In [2] A.F. Vasil'ev and T.I. Vasil'eva continued studying of  $c$ -supersoluble groups using the method of composition satellites. D. Robinson established the structural properties of  $c$ -supersoluble groups in the paper [3].

In [4] authors introduced the notion of  $Jc$ -supersoluble group that is local analogue of  $c$ -supersoluble group. Let  $J$  is a some class (possibly empty) of simple groups. We say that a group  $G$  is a  $J$ -group if the set  $\mathcal{K}_G$  of all composition factors of  $G$  is contained in  $J$ . Group  $G$  is called  $Jc$ -supersoluble if any chief  $J$ -factor of  $G$  is a simple group. A group  $G$  is called quasinilpotent ( $J$ -quasinilpotent) if for every chief factor ( $J$ -factor)  $H/K$  of  $G$  and every  $x \in G$ ,  $x$  induces an inner automorphism on  $H/K$ .

In the [5] some properties of the products of normal  $Jc$ -supersoluble subgroups have been established. In this report we studied the mutually permutable products of  $Jc$ -supersoluble groups. Recall [6, p. 149] that group  $G = HK$  is called the product of mutually permutable subgroups  $H$  and  $K$ , if  $H$  permutes with every subgroups of  $K$  and  $K$  permutes with every subgroups of  $H$ .

**Theorem 1.** *Let the group  $G = HK$  be the product of the mutually permutable subgroups  $H$  and  $K$ . If  $G$  is an  $Jc$ -supersoluble group, then  $H$  and  $K$  are both  $Jc$ -supersoluble groups.*

**Theorem 2.** *Let the group  $G = HK$  be the product of the mutually permutable subgroups  $H$  and  $K$ . If  $H$  is an  $Jc$ -supersoluble group and  $K$  is  $J$ -quasinilpotent, then  $G$  is an  $Jc$ -supersoluble group.*

**Theorem 3.** *Let the group  $G = HK$  be the product of the mutually permutable subgroups  $H$  and  $K$ . If  $H$  and  $K$  are  $Jc$ -supersoluble groups and  $G'$ , the derived subgroup of  $G$ , is  $J$ -quasinilpotent, then  $G$  is an  $Jc$ -supersoluble group.*

### References

- [1] V.A. Vedernikov, On some classes of finite groups. *Doklady Akademii nauk BSSR*. 2(10) (1988) 872-875.
- [2] A.F. Vasil'ev, T.I. Vasil'eva, On finite groups whose principal factors are simple groups. *Russian Mathematics (Izvestiya VUZ. Matematika)* 11 (1997) 10-14.
- [3] D.J.S. Robinson, The structure of finite groups in which permutability is a transitive relation. *J. Austral. Math. Soc.* 70 (2001) 143-149.
- [4] A.F. Vasil'ev, T.I. Vasil'eva, On finite groups with specified properties of chief series. *Mezhd. nauch. conf. «Discrete Mathematics, Algebra and Their Applications»*. (October 19–22, 2009, Minsk). *Tez. dokl. Mn., the Institute of Mathematics of NAS of Belarus* (2009). 12–14.
- [5] E.N. Myslovets, On products of normal generalized supersolvable subgroups of finite groups. *Proceedings of Francisk Scorina Gomel state university*. 6(75) (2012) 163–167.
- [6] A. Ballester-Bolinchés, R. Esteban-Romero, M. Asaad, Products of Finite Groups, Berlin/New York: Walter de Gruyter (2010) 334 p.