On Some Variants of the Post Correspondence Problem

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The Post correspondence problem (PCP) is well known and one of the most useful undecidable problems [1, 2]. The undecidability of PCP was shown in [3]. A large number of variants of the problem have been considered.

Some variants of PCP are decidable. In particular, we can mention Marked PCP [4], PCP for 2 rules [5], PCP over the unary alphabet [6], silly Post correspondence problem (SPCP) [2], Post embedding problem [7], and regular Post embedding problem [7]. Also, there are a number of polynomial formulations of PCP with bounded length of the word [6, 8–11].

It should be noted that SPCP is one of the simplest variants of PCP. In particular, SPCP can be solved in linear time. However, for group and commutative alphabets, we obtain the following results.

Theorem 1. SPCP *is* **NP***-complete for commutative alphabet and bounded length of the word.*

Theorem 2. SPCP *is* **NP***-complete for group alphabet and bounded length of the word.*

Theorem 3. SPCP *is undecidable for group alphabet and 5 rules.*

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