Groups, saturated with unitary groups of dimension three.

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Group G is saturated with a set of groups X, if every finite subgroup K of G is contained in a subgroup of G, which is isomorphic to a member of X [3].

Article [1] gives a description of periodic groups saturated by groups from a set $\mathfrak{N}$ ={$U_3(2^n)|n$ – arbitrary positive integer}. It was shown in [2] that a periodic Shunkov group, saturated by groups from a group set $\mathfrak{M}$ ={ $U_3(p^m)|$ p — an arbitrary prime number, n — narbitrary positive integer}, is isomorphic to $U_3(Q)$, where Q – is a suitable locally-finite field. The current work continues the investigations in that direction. Hereinafter, a symbol e will stand for the identity element of the group. The following results were obtained:

Theorem 1. *Let a periodic group G be saturated by groups from the set $\mathfrak{M}$ and S is the Sylow 2-subgroup of G takes one of the following forms:*

1. $S = \langle a^{2^n} = v^2 = 1, a^v = a^{2^{n-1}-1}\rangle$ — a semi-dihedral group.
2. $S = \langle a, w | a^{2^n} = b^{2^n} = w^2 = e, a^w = b, ab = ba\rangle$ — a wreath group.
3. S — is isomorphic to Sylow 2-subgroup $U_3(2^n)$.
4. S — is an infinite 2-group with a period of 4, nilpotency level equal 2, $S' = Z(S) = \Phi(S) = \Omega_1(S)$.
5. $S = (A \times B) \leftthreetimes \langle w\rangle$, where A — is an infinite locally-cyclic 2-group, $w^2 = e$, and $A^w = B$.
6. $S = AD$, where D is a finite subgroup of group S containing no wreath groups of order higher than 8, A — is an infinite locally-cyclic 2-group.

Theorem 2. *Shunkov group G, saturated with groups from the set $\mathfrak{M}$, has a periodic part $T(G)$, which is isomorphic to the group $U_3(Q)$, where Q is a suitable locally-finite field.*

References

[1] Lytkina D.V., Tuhvatullina L.R., Filippov K.A. Periodic groups, saturated with $U_3(2^n)$ // Algebra and Logic. 2008. V. 47. P. 288-306.

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[3] Shlepkin A.K. Conjugate double-primitive finite groups, consists finite unsolvable subgroups // Thesis of third international algebraic conference. Krasnoyarsk. 1993. P. 363.