# Special elements of the lattice of epigroup varieties

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A semigroup $S$ is called an *epigroup* if, for any element $x$ of $S$, some power of $x$ lies in some subgroup of $S$. On an epigroup, a natural unary operation named *pseudoinversion* may be defined (see [1, 2], for instance). This allows us to consider varieties of epigroups as algebras with the operations of multiplication and pseudoinversion.

We continue an examination of special elements of the lattice **EPI** of all epigroup varieties started in [3].

An element $x$ of a lattice $\langle L; \vee, \wedge \rangle$ is called *modular* if, for all $y, z \in L$, $(x \vee y) \wedge z = (x \wedge z) \vee y$ whenever $y \leq z$; *lower-modular* if, for all $y, z \in L$, $x \vee (y \wedge z) = y \wedge (x \vee z)$ whenever $x \leq y$; *distributive* if $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$ for all $y, z \in L$; *standard* if $(x \vee y) \wedge z = (x \wedge z) \vee (y \wedge z)$ for all $y, z \in L$; *neutral* if, for all $y, z \in L$, the sublattice of $L$ generated by $x$, $y$ and $z$ is distributive. *Upper-modular*, *codistributive* and *costandard* elements are defined dually to lower-modular, distributive and standard ones respectively.

Neutral, modular and upper-modular elements of the lattice **EPI** are considered in [3]. Here we investigate lower-modular, costandard and codistributive elements of **EPI**.

Put $\mathcal{ZM} = \mathrm{var}\{xy = 0\}$ and $\mathcal{SL} = \mathrm{var}\{x^2 = x, xy = yx\}$. We denote by $\mathcal{T}$ the trivial epigroup variety.

**Theorem 1.** *An epigroup variety $\mathcal{V}$ is a costandard element of the lattice* **EPI** *if and only if $\mathcal{V}$ is one of the varieties $\mathcal{T}$, $\mathcal{SL}$, $\mathcal{ZM}$ or $\mathcal{SL} \vee \mathcal{ZM}$.*

Recall that a variety is called 0-*reduced* if it may be given by identities of the form $w = 0$ only.

**Theorem 2.** *An epigroup variety $\mathcal{V}$ is a lower-modular element of the lattice* **EPI** *if and only if $\mathcal{V} = \mathcal{M} \vee \mathcal{N}$ where $\mathcal{M}$ is one of the varieties $\mathcal{T}$ or $\mathcal{SL}$ and $\mathcal{N}$ is a 0-reduced variety.*

Theorems 1 and 2 together with results of [3] imply that an element of **EPI** is costandard if and only if it is neutral, is modular whenever it is lower-modular, and is distributive if and only if it is standard.

An epigroup variety is called *strongly permutative* if it satisfies an identity of the type $x_1 x_2 \dots x_n = x_{1\pi} x_{2\pi} \dots x_{n\pi}$ where $\pi$ is a permutation on the set $\{1, 2, \dots, n\}$ with $1 \neq 1\pi$ and $n \neq n\pi$.

**Theorem 3.** *A strongly permutative epigroup variety $\mathcal{V}$ is a codistributive element of the lattice* **EPI** *if and only if $\mathcal{V} = \mathcal{G} \vee \mathcal{X}$ where $\mathcal{G}$ is a variety of Abelian groups and $\mathcal{X}$ is one of the varieties $\mathcal{T}$, $\mathcal{SL}$, $\mathcal{ZM}$ or $\mathcal{SL} \vee \mathcal{ZM}$.*

## References

[1] L. N. Shevrin, On theory of epigroups. I, II, Mat. Sbornik, **185** (1994), No. 8, 129–160; No. 9, 153–176.

[2] L. N. Shevrin, Epigroups, in: Structural Theory of Automata, Semigroups, and Universal Algebra, V. B. Kudryavtsev, I. G. Rosenberg (eds.), Springer, Dordrecht (2005), 331–380.

[3] V. Yu. Shaprynskiĭ, D. V. Skokov, B. M. Vernikov, Special elements of the lattice of epigroup varieties, Algebra Universalis, accepted; available at http://arxiv.org/abs/1408.0356v1.