Minimal generating systems and properties of sylow 2-subgroup of alternating group

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The aim of this paper is to research the structure of Sylow 2-subgroups and to construct a minimal generating system for such subgroups. Case of sylow subgroup where $p = 2$ is very special because it admits odd permutations, this case was not investigated in [1, 2]. There was a mistake in a statement about irreducebilyty that system of $k + 1$ elements for $Syl_2(A_{2^k})$ which was in abstract [3] in 2015 year. All undeclared terms are from [4]. A minimal system of generators for a sylow subgroup of A_n was found.

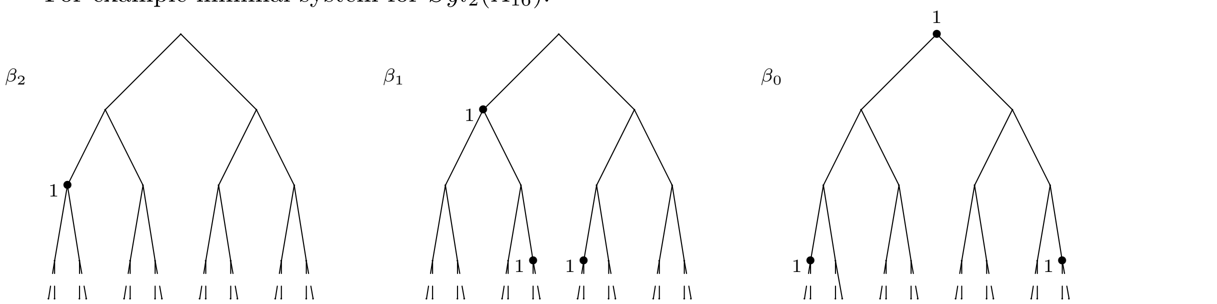
Let's denote by T_{k+1} a regular binary tree labeled by vertex. If the state in the vertex is non-trivial, then its label is 1, in other case it is 0. We denote by $v_{j,i}$ the vertex of L_j, which has the number i. An automorphism of T_{k+1} with non-trivial state in v_{1,i_1}, ... ,v_{1,i_j}, v_{2,j_2}, ... ,v_{k,k_m} is denoted by $\beta_{l_1,(i_1,\ldots i_J);l_1(i_1,\ldots i_J);\ldots;l_{k-1}(i_1,\ldots i_J)}$ where the index l_i is the number of level with non-trivial state. In parentheses after this numbers we denote a cortege of vertices of this level, where the non-trivial states in this automorphism are present. Denote by τ the automorphism, which has a non-trivial vertex permutation only in the first and the last vertices $v_{k,1}$ and $v_{k,2^k}$ of the last level L_k.

Lemma 1. *The set of elements from subgroup of AutT$_k$: $\alpha_{0,(1)}$, $\alpha_{1,(1)}$, $\alpha_{2,(1)}$, $\alpha_{k-2,(1)}$, τ, is system of generators for $Syl_2(A_{2^k})$.*

Lemma 2. *Oders of groups $\langle \alpha_{0,(1)}, \alpha_{1,(1)}, \alpha_{2,(1)}, \alpha_{k-2,(1)}, \tau \rangle$ and $Syl_2(A_{2^k})$ are equal to $2^{2^{k-2}}$.*

Main Theorem. *The set of elements from subgroup of AutT$_k$ $\beta_{0,(1);k,(1,2^k)}$, $\beta_{1,(1);k,(2^{k-1},2^{k-1}+1)}$, $\beta_{2,(1)}$, ... ,$\beta_{k-2,(1)}$ is minimal generators for a Sylow 2-subgroup of A_{2^k}.*

For example minimal system for $Syl_2(A_{16})$:



It was proved that the structure of sylow 2-subgroup of A_{2^k} is the following: $\wr_{i=1}^{k-1} C_2 \ltimes \prod_{i=1}^{2^k-1} C_2$, where we take C_2 as group of action on two elements and action is faithful.

References

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