On complements for $\mathfrak{F}$-residuals in finite groups

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In the theory of groups are well known results on the complementarity of an $\mathfrak{F}$-residual $G^{\mathfrak{F}}$ in a finite group G where $\mathfrak{F}$ is a local formation (see, for example, [1]). Using the properties of $\mathfrak{F}$-normalizers of G we obtain new results on the complementarity of $G^{\mathfrak{F}}$ by $\mathfrak{F}$-normalizers of the group G where $\mathfrak{F}$ is an ω-local Fitting formation and $\omega \subseteq \pi(\mathfrak{F})$.

We consider only finite groups. Not listed designations and definitions can be found in [1]. Let ω be a non-empty subset of the set of all primes $\mathbb{P}$, $f : \omega \cup \{\omega'\} \to \{$ formations of groups $\}$ is an ωF-function. A formation $\mathfrak{F} = (G : G/O_\omega(G) \in f(\omega')$ and $G/F_p(G) \in f(p)$ for all $p \in \omega \cap \pi(G))$ is called an ω-local formation with the ω-satellite f. Following [2] (see definition 2.6.1 [2]) we state the following definitions.

Definition 1. Let $\mathfrak{F}$ be a non-empty formation. A normal subgroup R of the group G is called an $\mathfrak{F}$-limited normal subgroup of G if $R \leq G^{\mathfrak{F}}$ and $R/R \cap \Phi(G)$ is a chief factor of the group G. A maximal subgroup M of G is called $\mathfrak{F}$-critical in G if $G = MR$ for some $\mathfrak{F}$-limited normal subgroup R of G.

Definition 2. Let $\mathfrak{F}$ be a non-empty ω-local formation. A subgroup H of the group G is called an $\mathfrak{F}$-normalizer of G if $H/\Phi(H) \cap O_{\omega'}(H) \in \mathfrak{F}$ and there exists a maximal chain $H = H_t \subset H_{t-1} \subset \cdots \subset H_1 \subset H_0 = G$ where H_i is an $\mathfrak{F}$-critical subgroup of H_{i-1} for each $i = 1, 2, \ldots, t$ and $0 \leq t$.

Theorem 1. *Let $\mathfrak{F}$ be a non-empty ω-local formation and let G be a group. Then there exists at least one $\mathfrak{F}$-normalizer H of the group G and $G = G^{\mathfrak{F}}H$.*

Theorem 2. *Let $\mathfrak{F}$ be a non-empty ω-local Fitting formation and let $G = A_1A_2\cdots A_n$ be a group where A_i is a subnormal subgroup of G for each $i = 1, 2, \ldots, n$ and $\omega \subseteq \pi = \pi(\mathfrak{F})$. If a $\mathfrak{F}$-residual of A_i is ω-soluble and for every $p \in \omega$ Sylow p-subgroups of $A_i^{\mathfrak{F}}$ is abelian for each $i = 1, 2, \ldots, n$ then every $\mathfrak{F}$-normalizer of G is an ω-complement for $G^{\mathfrak{F}}$ in G.*

Corollary 1. *Let $\mathfrak{F}$ be a local non-empty Fitting formation and let $G = A_1A_2\cdots A_n$ be a group where A_i is a subnormal subgroup of G for each $i = 1, 2, \ldots, n$. If an $\mathfrak{F}$-residual $A_i^{\mathfrak{F}}$ is $\pi(\mathfrak{F})$-soluble for every $i = 1, 2, \ldots, n$ and its Sylow p-subgroups are abelian for all $p \in \pi(\mathfrak{F})$ then each $\mathfrak{F}$-normalizer of G is the complement for $\mathfrak{F}$-residual $G^{\mathfrak{F}}$ in G.*

Corollary 2. *Let $\mathfrak{F}$ be a local non-empty Fitting formation and let $G = A_1A_2\cdots A_n$ be a group where A_i is a subnormal subgroup of G for each $i = 1, 2, \ldots, n$. If $\mathfrak{F}$-residual $A_i^{\mathfrak{F}}$ is abelian for every $i = 1, 2, \ldots, n$ then each $\mathfrak{F}$-normalizer of G is the complement for $\mathfrak{F}$-residual $G^{\mathfrak{F}}$ in G.*

Theorem 3 *Let $\mathfrak{F}$ be a non-empty ω-local formation, let G be a group and let ω_1 be a set of all primes $p \in \omega$ for which $G^{\mathfrak{F}}$ has an abelian Sylow p-subgroup. Then $G^{\mathfrak{F}}$ has an ω_1-complement in any extension of G.*

Theorem 4 *Let $\mathfrak{F}$ be a non-empty ω-local formations, let Γ be an extension of the group G and let $\omega_1 = \{p \in \mathbb{P} |\ p$ divides $(| \Gamma : G^{\mathfrak{F}} |, | G^{\mathfrak{F}} |)\}$. If $\omega_1 \subseteq \omega$ and a Sylow p-subgroup of $G^{\mathfrak{F}}$ is abelian for each $p \in \omega_1$, then $G^{\mathfrak{F}}$ has a complement in the group Γ.*

References

[1] L. A. Shemetkov, Formations of finite groups. *M.: Nauka* (1978)

[2] W. Guo, The theory of classes of groups. *Beijing, New York: Kluwer Academic Publishers Science Press* (2000)