The eigenfunctions with the minimum support of the cubic distance-regular graphs

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Let $G = (V, E)$ be an undirected graph without loops and multiple edges with the vertex set $V = \{1, 2, \ldots, n\}$ and the edge set E. G is *regular* if each vertex has the same number k of the neighbours. The parameter k is called the *degree* of the graph. For any vertices $v, u \in V$ the *distance* $d(v, u)$ is the number of edges in the shortest path that connects them. By $G_i(v)$ we denote the set of the vertices that are at distance i from v. A connected graph G is called *distance-regular* if it is regular of degree k and for any two vertices $v, u \in V$ at distance $i = d(v, u)$, there are precisely c_i neighbours of u in $G_{i-1}(v)$ and b_i neighbours of u in $G_{i+1}(v)$. The numbers $b_i, c_i, a_i = k - b_i - c_i$ are called the *intersection numbers* of G.

Consider the adjacency matrix A of order n, defined as following:

$$A_{ij} = \begin{cases} 1, & \text{when } ij \in E \\ 0, & \text{when } ij \notin E \end{cases}$$

For a matrix A let $\Lambda = \{\lambda_1, \ldots, \lambda_t\}$ be the set of its eigenvalues. If $f = (f_1, \ldots, f_n)$ is a function on the graph vertices that satisfies the equation $Af = \lambda f$, we call it an *eigenfunction* of the graph G corresponding to the eigenvalue λ. The support *supp*(f) of the function f is the set of its non-zero coordinates, i.e. *supp*$(f) = \{i \mid f_i \neq 0\}$. We are interested in finding the eigenfunctions with the supports of minimum cardinality.

In the current work we study the distance-regular graphs of the degree $k = 3$. It is known [1] that up to isomorphism there are only 13 of them: K_4, $K_{3,3}$, the Petersen graph, the cube, the Heawood graph, the Pappus graph, the Coxeter graph, the Tutte-Coxeter graph, the dodecahedron, the Desargues graph, the Foster graph, the Tutte 12-cage, the Biggs-Smith graph. For all of them, except for the last two graphs, we found the cardinalities of the minimum supports of the eigenfunctions over the field $\mathbb{R}$ and classified their structures for all the eigenvalues.

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References

[1] N.L. Biggs, A.G. Boshier, J. Shawe-Taylor, Cubic distance-regular graphs // J. London Math.Soc. 1986. Vol. 33, no. 2. P. 385–394.