About some products K-$\mathbb{P}$-subnormal subgroups of finite groups

Artem Vegera
F. Skorina Gomel State University, Gomel, Belarus

We consider only finite groups. In 1978 O. Kegel [1] proposed the concept of K-$\mathfrak{F}$-subnomal subgroup.

Let $\mathfrak{F}$ be a non-empty hereditary formation. A subgroup H of a group G is called K-$\mathfrak{F}$-subnormal ($\mathfrak{F}$-reachable [1]) subgroup of G (denoted H K-$\mathfrak{F}$-sn G), if there is a chain of subgroups $H = H_0 \subseteq H_1 \subseteq \cdots \subseteq H_n = G$ such that or $H_{i-1} \lhd H_i$, or $H_i^{\mathfrak{F}} \subseteq H_{i-1}$, for $i = 1, \ldots, n$.

In papers [2] and [3] A.F.Vasil'ev, T.I.Vasil'eva, V.N.Tyutyanov introduced the definitions of $\mathbb{P}$-subnormality and K-$\mathbb{P}$-subnormality for subgroups respectively.

Definition 1 [3]. A subgroup H of group G is called K-$\mathbb{P}$-subnormal in G (denoted H K-$\mathbb{P}$-sn G), if there is a chain of subgroups $H = H_0 \subseteq H_1 \subseteq \cdots \subseteq H_n = G$ such that either H_{i-1} is normal in H_i or $|H_i : H_{i-1}|$ is prime for every $i = 1, \ldots, n$.

Let $\mathfrak{U}$ be the formation of all supersoluble groups, then every K-$\mathfrak{U}$-subnormal subgroup of G is K-$\mathbb{P}$-subnormal in G. The converse assertion fails to hold in general.

In [3] authors studied the properties of products of groups $G = AB$ where A and B are K-$\mathbb{P}$-subnormal in G. In the present article we continue investigations of [3] in the case if a group G is the product of its pairwise permutable subgroups $G_1, G_2, \ldots, G_n$, ie $G = G_1G_2 \ldots G_n$ and $G_iG_j = G_jG_i$ for all integers i and j with $i, j \in \{1, 2, \ldots n\}$.

Definition 2 [3]. A group G is called $\overline{\mathrm{w}}$-supersoluble if every Sylow subgroup of G is K-$\mathbb{P}$-subnormal in G.

Theorem 1. *Let $G = G_1G_2 \ldots G_n$ be the product of its pairwise permutable Ore dispersive subgroups G_1 $G_2, \ldots, G_n$, subgroups G_i K-$\mathbb{P}$-sn G_iG_j and G_j K-$\mathbb{P}$-sn G_iG_j for each $i, j \in \{1, 2, \ldots n\}$. Then G is Ore dispersive.*

Theorem 2. *Let $G = G_1G_2 \ldots G_n$ be the product of its pairwise permutable nilpotent subgroups G_1 $G_2, \ldots, G_n$, subgroups G_i K-$\mathbb{P}$-sn G_iG_j and G_j K-$\mathbb{P}$-sn G_iG_j for each $i, j \in \{1, 2, \ldots n\}$. Then G is $\overline{\mathrm{w}}$-supersoluble.*

Recall [2] a generalized commutant of a group G is called the smallest normal subgroup N of G such that G/N is a group with abelian Sylow subgroups.

Theorem 3. *Let $G = G_1G_2 \cdots G_n$ be the product of its pairwise permutable $\overline{\mathrm{w}}$-supersoluble subgroups $G_1, G_2, \ldots, G_n$, subgroups G_i K-$\mathbb{P}$-sn G_iG_j and G_j K-$\mathbb{P}$-sn G_iG_j for each $i, j \in \{1, 2, \ldots n\}$. If the generalized commutant of group G is nilpotent, then G is $\overline{\mathrm{w}}$-supersoluble.*

References

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