Upper-modular and related elements of the lattice of commutative semigroup varieties

Boris Vernikov
Institute of Mathematics and Computer Science, Ural Federal University, Yekaterinburg, Russia

We denote by **SEM** the lattice all semigroup varieties and by **Com** the sublattice of **SEM** consisting of all commutative varieties. During last decade, about 15 articles devoted to examination of special elements of different types in these two lattices were appeared. The results obtained here are overviewed in the recent article [1]. Special elements of eight types were considered in the mentioned articles, namely neutral, standard, costandard, distributive, codistributive, modular, lower-modular and upper-modular elements (the definitions see in [1]). In the lattice **SEM**, neutral, standard, costandard, distributive or lower-modular elements are completely described, and a significant results concerning codistributive, modular or upper-modular elements were obtained. In the lattice **Com**, neutral, standard, distributive or lower-modular elements were completely determined, and a significant results about modular elements were proved. But there no any information about costandard, codistributive or upper-modular elements in **Com** up to the recent time. The following two theorems give a complete description of these elements.

Theorem 1. *For a commutative semigroup variety $\mathcal{V}$, the following are equivalent:*

a) *$\mathcal{V}$ is an upper-modular element in the lattice* **Com***;*

b) *$\mathcal{V}$ is a codistributive element in the lattice* **Com***;*

c) *one of the following holds:*

(i) *$\mathcal{V}$ is the variety of all commutative semigroups;*

(ii) *$\mathcal{V} = \mathcal{M} \vee \mathcal{N}$ where $\mathcal{M}$ is either the trivial variety $\mathcal{T}$ or the variety of semilattices $\mathcal{SL}$, and $\mathcal{N}$ is a commutative variety with the identities $x^2yz = 0$ and $x^2y = xy^2$;*

(iii) *$\mathcal{V} = \mathcal{G} \vee \mathcal{M} \vee \mathcal{N}$ where $\mathcal{G}$ is a variety of periodic Abelian groups, $\mathcal{M}$ is one of the varieties $\mathcal{T}$, $\mathcal{SL}$ or $\mathrm{var}\{x^2 = x^3, xy = yx\}$, and $\mathcal{N}$ is a commutative variety with the identity $x^2y = 0$.*

Theorem 2. *A commutative semigroup variety $\mathcal{V}$ is a costandard element in the lattice* **Com** *if and only if one of the claims* (i) *or* (ii) *of Theorem* 1 *holds.*

References

[1] B.M.Vernikov, Special elements in lattices of semigroup varieties // *Acta Sci. Math.* (*Szeged*). 2015. Vol. 81. P. 79-109.