On intersections of nilpotent subgroups in finite groups with the socle isomorphic to $\Omega_8^+(2)$

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Let G is a finite group with the socle $Soc(G)$ isomorphic to $\Omega_8^+(2)$. Then (see [1]) $Out(\Omega_8^+(2)) \cong \Sigma_3$ and $Soc(G)$ contains a parabolic subgroup P such that P is normalized by an involution τ which induces the graph automorpihism on $Soc(G)$ and Levi subgroup of P is isomorphic to $L_3(2)$.

For subgroups A and B of G, denote by $M_G(A,B)$ the set of minimal under the inclusion intersections $A \cap B^g$ where $g \in G$ and by $m_G(A,B)$ the set of minimal under the order elements from $M_G(A,B)$. Set $Min_G(A,B) = \langle M_G(A,B) \rangle$ and $min_G(A,B) = \langle m_G(A,B) \rangle$.

The following two theorems are proved.

Theorem 1. *Let G be a finite group with $Soc(G) \cong \Omega_8^+(2)$ and $S \in Syl_2(G)$. If $min_G(S,S) \neq 1$ then $G = Soc(G)\langle\tau\rangle$ and $min_G(S,S) = O_2(P)\langle\tau\rangle$.*

Theorem 2. *Let G be a finite group with $Soc(G) \cong \Omega_8^+(2)$, $S \in Syl_2(G)$, A and B be nilpotent subgroups of G. Then the following conditions are equivalent:*

(1) *$A \cap B^g \neq 1$ for any $g \in G$;*

(2) *$min_G(A,B) \neq 1$;*

(3) *$Min_G(A,B) \neq 1$;*

(4) *$G = Soc(G)\langle\tau\rangle$, A and B are conjugated to some subgroups A^g and B^h of S such that $A^g \cap B^h \geq min_G(S,S)$.*

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References

[1] **J. Conway, R. Curtis, S. Norton, R. Parker, R. Wilson**. Atlas of finite groups. Clarendon Press: Oxford, 1985.