Ural Federal University named after the first President of Russia B.N. Yeltsin
N.N. Krasovskii Institute of Mathematics and Mechanics UB RAS
Sobolev Institute of Mathematics SB RAS

Groups and Graphs, Algorithms and Automata

Yekaterinburg, Russia, August, 9-15, 2015

Abstracts of the International Conference and PhD Summer School in honor of the 80th Birthday of Professor Vyacheslav A. Belonogov and of the 70th Birthday of Professor Vitaly A. Baransky

Yekaterinburg – 2015

512.54, 519.17, 519.713, 510.53

BBK 22.17

G73

GROUPS AND GRAPHS, ALGORITHMS AND AUTOMATA, 2015: Abstracts of the International Conference and PhD Summer School in honor of the 80th Birthday of Professor Vyacheslav A. Belonogov and of the 70th Birthday of Professor Vitaly A. Baransky. Yekaterinburg: UrFU Publishing house, 2015. – p.109

ISBN 978-5-86134-169-1

Published by
UrFU Publishing house,
ul. Turgeneva 4, 620000,
Yekaterinburg, 2015.

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Booklet Cover Designer
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Print run - 120 - Not for sale

ISBN 978-5-86134-169-1

Contents

General Information

The International Conference and PhD Summer School "Groups and Graphs, Algorithms and Automata" is organized by the N.N. Krasovskii Institute of Mathematics and Mechanics of UB RAS, Ural Federal University named after the first President of Russia B.N. Yeltsin and OOO "Адаптивные решения" (Adaptive Solutions Ltd.).

The conference is dedicated to Professor Vyacheslav A. Belonogov in the occasion of his 80th birthday and to Professor Vitaly A. Baransky in the occasion of his 70th birthday.

The conference aims to cover all branches of group theory, graph theory, automata and formal language theory and algorithm theory. The scientific program consists of Minicourses, Plenary and Contributed talks. The official language of the conference is English.

The Conference venue is recreation area Ivolga near Yekaterinburg, August, 9-15, 2015.

Scientific committee:
Alexander Makhnev (*chair*), Alexander Gavrilyuk, Lev Kazarin, Anatoly Kondrat'ev, Elena Konstantinova, Natalia Maslova, Viktor Mazurov, Dmitrii Paduchikh, Danila Revin, Vladimir Trofimov, Andrey Vasil'ev, Mikhail Volkov, Viktor Zenkov.

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Conference Program

August 9, 2015

09:00 - 12:00	Registration: *Ural Federal University named after the first President of Russia B.N. Yeltsin: Hall, 51, Lenina Pr., Yekaterinburg*
12:30 - 13:30	*Getting Ivolga*
14:00 - 15:00	*Lunch*
15:30 - 15:45	*Official Welcome*
Plenary Talks	
Special session dedicated to Professor V. Baransky in the occasion of his 70th birthday	
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16:00 - 16:50	V. Baransky (join work with T. Nadymova and T. Senchonok): *The lattice of graphical partitions*
16:50 - 17:40	A. Makhnev: *Koolen problem for* $t = 5$
18:00 - 19:00	*Football*
19:00 - 20:00	*Dinner*

August 10, 2015

08:30 - 09:45	*Breakfast*
Plenary Talks	
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10:50 - 11:40	J. Koolen (join work with Z. Qiao, A. Gavrilyuk and J. Park): *On recent progress of 2-walk-regular graphs*
11:40 - 12:10	*Coffee break*
12:10 - 13:00	V. Kabanov (join work with L. Shalaginov): *On some classes of Deza graphs*
13:00 - 14:00	*Lunch*
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15:20 - 16:10	K. Kutnar: *Minicourse I, Lecture 2*
16:10 - 16:40	*Coffee break*
Contributed Talks	
19:00 - 20:00	*Dinner*
20:00 - 22:00	*Problem solving / Sports*

August 11, 2015

08:30 - 09:45 *Breakfast*
Plenary Talks and Minicourses
10:00 - 10:50 S. Goryainov (join work with A. Gavrilyuk and L. Shalaginov): *On Deza circulants*
10:50 - 11:40 E. Vdovin: *Minicourse IV, Lecture 1*
11:40 - 12:10 *Coffee break*
12:10 - 13:00 T. Pisanski: *Minicourse II, Lecture 1*
13:00 - 14:00 *Lunch*
Minicourses
14:30 - 15:20 D. Marušič: *Minicourse I, Lecture 3*
15:20 - 16:10 K. Kutnar: *Minicourse I, Lecture 4*
16:10 - 16:40 *Coffee break*
Contributed Talks
19:00 - 20:00 *Dinner*
20:00 - 22:00 *Problem solving / Sports*

August 12, 2015

08:30 - 09:45 *Breakfast*
Plenary Talks
Special session dedicated to Professor V. Belonogov in the occasion of his 80th birthday
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10:15 - 11:05 V. Belonogov: *Character theory and abstract structure of finite groups*
11:05 - 11:55 L. Kazarin: *Group factorizations, graphs and related topics*
11:55 - 12:25 *Coffee break*
12:25 - 13:15 B. Amberg: *Products of groups which contain abelian subgroups of finite index*
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14:30 - 15:20 A. Kondrat'ev: *On prime graphs of finite groups*
15:20 - 16:10 N. Maslova (join work with A. Kondrat'ev and D. Revin): *On the pronormality of subgroups of odd indices in finite simple groups*
16:10 - 16:40 *Coffee break*
16:40 - 17:30 V. Levchuk (join work with O. Kravtsova): *Problems on structure of finite quasifields and projective translation planes*
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19:00 *Conference Dinner*

August 13, 2015

08:30 - 09:45 *Breakfast*
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10:50 - 11:40 E. Vdovin: *Minicourse IV, Lecture 2*
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12:10 - 13:00 T. Pisanski: *Minicourse II, Lecture 2*
13:00 - 14:00 *Lunch*
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16:10 - 16:40 *Coffee break*
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20:00 - 22:00 *Problem solving / Sports*

August 14, 2015

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15:20 - 16:10 K. Kutnar: *Minicourse I, Lecture 8*
16:10 - 16:40 *Coffee break*
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20:00 - 22:00 *Problem solving / Sports*

August 15, 2015

08:30 - 09:45 *Breakfast*
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10:50 - 11:40 N. Timofeeva: *Minicourse III, Lecture 2*
11:40 - 12:10 *Coffee break*
12:10 - 13:00 T. Pisanski: *Minicourse II, Lecture 4*
13:00 - 14:00 *Lunch*
Plenary Talks
14:30 - 15:20 V. Trofimov: *Some problems concerning vertex-symmetric graphs*
15:30 - 15:50 *Closing*
17:00 *Leaving Ivolga*

Abstracts

Abstracts of Minicourses, Plenary and Contributed talks are listed alphabetically with respect to Presenting Author

Minicourses

Minicourse I: Graphs and their automorphism groups

Lecturers:
Klavdija Kutnar
University of Primorska, Koper, Slovenia

Dragan Marušič
University of Primorska, Koper, Slovenia

In mathematics we usually tend to study structures that admit a certain degree of symmetry. In graphs the degree of symmetry is given by the automorphism group which is the group of all adjacency preserving permutations of its vertex set.

In this course we introduce various families of graphs with a rather large degree of symmetry such as vertex-transitive graphs, edge-transitive graphs and arc-transitive graphs. We review some of the methods for constructing such graphs and present some results from the rich theory that has developed in the last few decades. We also present some open problems in the area.

The course contains 8 lectures.

This course is organized in the frame of the international cooperation between Slovenia and Russia in 2014-2015.

Minicourse II: Symmetries in Graphs with Python and Sage

Lecturer:
Tomaž Pisanski
University of Ljubljana, Ljubljana, Slovenia
University of Primorska, Koper, Slovenia

In this course we will learn basics of Python and Sage that will enable participants to start exploring non-trivial questions about symmetries of graphs. We will construct some bi-Cayley graphs, such as Haar graphs, rose-window graphs, I-graphs and their generalizations. We will also analyze some existing censuses of graphs and related structures, such as maps and polytopes.

Each participant is expected to posses basic skills of computer programming and have his or her own lap-top available.

Each registered participant will receive a handout, including relevant references and tutorials, prior to the beginning of this minicourse.

A list of questions, ranging from simple exercises that will enable participants to recall the learned skills, to non-trivial mathematical problems will be distributed.

The course contains 4 lectures.

This course is organized in the frame of the international cooperation between Slovenia and Russia in 2014-2015.

Minicourse III: Monstrous Moonshine

Lecturer:
Nadezhda Timofeeva
Yaroslavl P. Demidov State University, Yaroslavl, Russia

A starting point was the paper of 1979 by J.H. Conway and S.P. Norton entitled “Monstrous Moonshine”. It comprises of several seeming-coincidences relating the Monster group (in that time its existence was only conjectured) to modular forms. Since this original paper many more connections of modular forms to sporadic simple groups were discovered. They all are collectively referred to as Moonshine.

In 1998 R. Borcherds won the Fields medal in part for his work where he proved the original conjectures of J.H. Conway and S.P. Norton. The proof opened connections of the representation theory and the theory of modular forms to mathematical physics.

In my lectures I will try to explain the basic notions and to describe the key moments of the Moonshine in its classical version. If the time permit, I will sketch some generalisations and say several words about open problems.

The course contains 2 lectures.

Minicourse IV: Existence and conjugacy of Hall subgroups. Contemporary progress and open problems

Lecturer:
Evgeny Vdovin
Sobolev Institute of Mathematics SB RAS, Novosibirsk, Russia
Novosibirsk State University, Novosibirsk, Russia

In the lectures we plan to discuss general methods for answering to the following problems: whether given finite group possesses a π-Hall subgroup for a set of primes π, and how many classes of conjugate π-Hall subgroups the group has.

One of the main technical tool is the notion of a group of induced automorphisms and the inclusion to the wreath product with this group (see [1], and theorem 3 from this paper). We recommend the attendants to read paper [2] also (at least the main part without Appendix).

The course contains 2 lectures.

Reference

[1] E. P. Vdovin, Groups of induced automorphisms and their application to studying the existence problem for Hall subgroups // Algebra and Logic. 2014. Vol. 53, no. 5. P. 418–421. Doi:10.1007/s10469-014-9301-x.

[2] E. P. Vdovin, D. O. Revin, Theorems of Sylow type // Russian Math. Surveys. 2011. Vol. 66, no. 5. P. 829–870.

Minicourse V: Synchronizing finite automata: a problem everyone can understand but nobody can solve (so far)

Lecturer:
Mikhail Volkov
Institute of Mathematics and Computer Science, Ural Federal University, Yekaterinburg, Russia

Most current mathematical research, since the 60s, is devoted to fancy situations: it brings solutions which nobody understands to questions nobody asked (quoted from Bernard Beauzamy in [1]). This provocative claim is certainly exaggerated but it does reflect a really serious problem: a communication barrier between mathematics (and exact science in general) and human common sense. The barrier results in a paradox: while the achievements of modern mathematics are widely used in many crucial aspects of everyday life, people tend to believe that today mathematicians do "abstract nonsense" of no use at all. In most cases it is indeed very difficult to explain to a non-mathematician what mathematicians work with and how their results can be applied in practice. Fortunately, there are some lucky exceptions, and one of them has been chosen as the present course's topic. It is devoted to a mathematical problem that was frequently asked by both theoreticians and practitioners in many areas of science and engineering. The problem, usually referred to as the synchronization problem, can be roughly described as the task of determining the simplest way to restore control over a device whose current state is not known - think of a satellite which loops around the Moon and cannot be controlled from the Earth while "behind" the Moon. While easy to understand and practically important, the synchronization problem turns out to be surprisingly hard to solve even for finite automata that constitute the simplest mathematical model of real-world devices. This combination of transparency, usefulness and unexpected hardness should, hopefully, make the course interesting for a wide audience.

Among other things, the course will present a recent major advance in the theory of synchronizing finite automata: Avraam Trahtman's proof of the so-called Road Coloring Conjecture by Adler, Goodwyn, and Weiss. The conjecture that admits a formulation in terms of recreational mathematics arose in symbolic dynamics and has important implications in coding theory. The proof is elementary in its essence but clever and enjoyable.

The course contains 3 lectures.

Reference

[1] B. Beauzamy, Real life mathematics // Irish Math. Soc. Bull. 2002. Vol. 48. P. 43–46.

Plenary Talks

On products of groups which contain almost abelian subgroups

Bernhard Amberg
Johannes Gutenberg-Universität Mainz, Mainz, Germany

Let the group $G = AB$ be the product of two subgroups A and B, i. e. $G = \{ab \mid a \in A, b \in B\}$. If A and B are abelian, then G is metabelian by a well-known theorem of N. Itô (see for instance [1]). This raises the question whether every group $G = AB$ with abelian-by-finite subgroups A and B is metabelian-by-finite ([1], Question 3), or at least soluble-by-finite. However, this seemingly simple question is very difficult to attack. A positive answer was previously given under additional requirements, for instance for linear groups G by Ya. Sysak and for residually finite groups G by J. Wilson, see [1]. Furthermore, N. S. Chernikov proved that every group $G = AB$ with central-by-finite subgroups A and B is soluble-by-finite (see [1]).

It is natural first to consider groups $G = AB$ where the two factors A and B have abelian subgroups with small index, in particular less or equal 2. In the talk some results obtained recently by Lev Kazarin, Yaroslav Sysak and myself in the case that enough involutions are present will be presented.

For example the following holds.

Theorem. *Let the group $G = AB$ be the product of two subgroups A and B each of which is either abelian or generalized dihedral. Then G is soluble.*

Here a group A is called generalized dihedral if it contains an abelian subgroup X of index 2 and an involution inverting every element of X. Clearly A is a semidirect product $A = X \rtimes < a >$ of the abelian group X with a group $< a >$ of order 2 such that $x^a = x^{-1}$ for every $x \in X$. Obviously dihedral groups and locally dihedral groups are also generalized dihedral.

Reference

[1] B. Amberg, S. Franciosi, F. de Giovanni, Products of groups. Oxford: Clarendon Press, 1992.

The lattice of graphical partitions

Vitaly Baransky
Institute of Mathematics and Computer Science, Ural Federal University, Yekaterinburg, Russia

This is joint work with Tatyana Nadymova and Tatiana Senchonok

A *partition* is a sequence of nonnegative integers (the parts) in nonincreasing order (we will disregard trailing zeroes).

A *graphical partition* is a partition whose parts can be interpreted as the degrees of the vertices of some simple (undirected) graph.

We show that, for a given integer n,
– all graphical partitions,
– all graphical partitions of lengths less than or equal to n,
– all graphical partitions of length n
form the lattices GPL, $GPL(n)$, $GPLzf(n)$ ordered by dominance.

We show that the lattice GPL is a lower subsemilattice of the lattice NPL of all partitions ordered by dominance, but GPL is not a sublattice of the lattice NPL.

We establish that the set of all graphical partitions of $2m$ is an order ideal of the lattice of all partitions of $2m$. We find, for a given integer m, all maximal graphical partitions in the lattice of all partitions of $2m$.

We also present a new algorithm, which, for a given integer n, generates all graphical partitions of lengths less than or equal to n. Our algorithm can generate graphical partitions without generating any nongraphical partitions.

Reference

[1] T. Brylawski, The lattice of integer partitions // Discrete Math. 1973. Vol. 6, no. 3. P. 201–219.

[2] G. Sierksma, H. Hoogeveen, Seven criteria for integer sequences being graphic // J. Graph Theory. 1991. Vol. 15, no. 2. P. 223–231.

[3] A. Kohnert, Dominance order and graphical partitions // The Electron. J. Comb. 2004. Vol. 11, no. 4. P. 1–17.

[4] J.M. Burns, The number of degree sequences of graphs // MIT. 2007. P. 1–60.

[5] V.A. Baransky, T.A. Koroleva, The lattice of partitions of an integer // Doklady Math. 2008. Vol. 77, no. 1. P. 72–75.

[6] A. Iványi, L. Lucz, G. Gombos, T. Matuszka, Parallel enumeration of degree sequences of simple graphs II // Acta Univ. Sapientiae, Informatica. 2013. Vol. 5, no. 2. P. 245–270.

Character theory and abstract structure of finite groups

Vyacheslav Belonogov
N.N. Krasovskii Institute of Mathematics and Mechanics UB RAS, Yekaterinburg, Russia

This talk is a short survey of some results from the character theory of finite groups which are used for the study of the abstract structure of groups. In particular, some results of the author are discussed. We consider the following themes.

1. Some notation and elementary definitions.
2. Character table of a group.
3. Interactions and D-blocks.
4. Zeroes in the character table.
5. Characterization of groups by active fragments of the character table.
6. Semiproportional characters.

1. Further, G is a finite group and $\mathbb{C}$ is the field of all complex numbers. If $g \in G$ then $C_G(g)$ is the centralizer of g in G, $g^G := \{x^{-1}gx \mid x \in G\}$ is the conjugacy class of G containing g, and $k(G)$ is the number of conjugacy classes of G. We remember concepts: a *representation of* G *over a field* F; the *degree* of a representation; the *character* of a representation; *reducible* and *irreducible* representations; the *kernel* of a representation. The writing $D \trianglelefteq G$ denotes that D is a normal subset of G (i. e. the union of some conjugacy classes of G). Majority of necessary to us concepts and results may be find in [1–3].

2. A *character* (*irreducible character*) *of* G is a character of some representation (respectively, irreducible representation) of G over $\mathbb{C}$. Irr(G) denotes the set of all irreducible characters of G. Then $|\mathrm{Irr}(G)| = k(G)$. If $\mathrm{Irr}(G) = \{\chi_1, \chi_2, \ldots, \chi_k\}$ and $\mathrm{Cl}(G) = \{{g_1}^G, {g_2}^G, \ldots, {g_k}^G\}$, where $k = k(G)$, then ($k \times k$-matrix) $\mathrm{X}(G) = (\chi_i(g_j))$ ($k \times k$-matrix) is a *character table* of G (X is the Greek Chi). The *orthogonality relations* in $\mathrm{X}(G)$ are significant.

Problem 1. *To investigate the interdependency of the properties of the character table of a group and the abstract structure of this group.*

2a. From G to $\mathrm{X}(G)$: For any given group G may be constructed $\mathrm{X}(G)$ (see [2, theorem 10]).

2b. From $\mathrm{X}(G)$ to G: The size of the table $\mathrm{X}(G)$ is very small with respect to $|G|$ (examples are given) in order that determinate G (up to isomorphism) by $\mathrm{X}(G)$. $X(D_8) = X(Q_8)$ almost $D_8 \ncong Q_8$. Nevertheless, it is possible recognize many properties of G from $\mathrm{X}(G)$. There exist some groups that may be completely reconstructed (up to isomorphism) by their character tables. In particular, this property have groups S_n [4] and A_n [5].

3. We remind the concepts of *interaction* and *D-block* (where $D \trianglelefteq G$) introduced in [6] (see also [3, chapter 3, sect. A]) and discuss some appropriate results of the author (in particular, [10]). The concept of D-block generalizes the classical concept of p-block (where p is a prime number): *If* D *is the set* $G_{p'}$ *of all* p'*-elements of* G*, then the concept of* D*-block coincides with the concept of* p*-block.* We discuss an effective method (from [7]) for calculating p-blocks of finite groups which is based on using of D-blocks for some p-sections D of a given group.

4. For applications of the character theory to study the abstract structure of groups, results on existence and disposition of zeros in $\mathrm{X}(G)$ are important. Here we give some examples of such results. In particular, one of such results (see [8]) has Corollary: if $\mathrm{X}(G)$ has a zero submatrix $O_{s\times t}$ then $s + t \leq k(G) - 1$. A zero submatrix $O_{s\times t}$ of $\mathrm{X}(G)$ with $s + t = k(G) - 1$ is called the *extremal zero fragment* of $\mathrm{X}(G)$. The following problem is not solved till now.

Problem 2. *To investigate groups* G *such that* $\mathrm{X}(G)$ *has an extremal zero fragment.*

5. Let $D \trianglelefteq G$, $\Phi \subseteq \mathrm{Irr}(G)$ and $\mathrm{X}(\Phi, D)$ is the submatrix of $\mathrm{X}(G)$ lying on intersections of rows corresponding to characters in Φ and columns corresponding to classes in D. If D is interact with Φ then the matrix $\mathrm{X}(\Phi, D)$ is called an *active fragment* of $\mathrm{X}(G)$ or an active fragment of G. We shall discuss some established by the author characterizations of finite groups (in particular, J_1 [9], $PSL_2(q)$ and $Sz(q)$) by their active fragments.

6. Functions φ and ψ from a set M in the field $\mathbb{C}$ is called *semiproportional*, if they are not proportional and there is a subset H in M such that $\varphi|_M$ is proportional to $\psi|_M$ and $\varphi|_{S\setminus M}$ is proportional to $\psi|_{S\setminus M}$ ($\varphi|_M$ denotes the restriction of φ to M). In particular, we may speak on semiproportional characters of a group, on semiproportional rows and on semiproportional column of $X(G)$. For brevity, two conjugacy classes of G we shall call *semiproportional* if corresponding to them columns of $X(G)$ are semiproportional. We discuss (see, in particular, [11–15]) the following conjectures.

Conjecture 1 (Semiproportional Characters Conjecture). *If φ and ψ are semiproportional irreducible characters of a finite group then $\varphi(1) = \psi(1)$.*

Conjecture 2 (Semiproportional Classes Conjecture). *If g^G and h^G are semiproportional conjugacy classes of a finite group G then the cardinality of one from this classes divides the cardinality of other.*

We discuss also some results connected with following theorem (the concluding result is obtained in [13]).

Theorem. *Finite alternating groups have no semiproportional irreducible characters.*

Reference

[1] I. M. Isaacs, Character theory of finite groups. N. Y.: Acad. Press. 1978.

[2] V. A. Belonogov, A. N. Fomin, Matrix reprezentatios in the theory of finite groups. M.: Nauka. 1976. (in Russian)

[3] V. A. Belonogov, Reprezentatios and characters in the theory of finite groups. Sverdlovsk: Ural Branch of AS USSR, 1990. (in Russian).

[4] H. Nagao, On the groups with the same table of characters as symmetric groups // J. Inst. Polytech. Osaka City Univ. Ser. A . 1957. Vol. 8, no. 1. P. 1–8.

[5] T. Oyama, On the groups with the same table of characters as alternating groups // Osaka J. Math. 1964. Vol. 1, no. 1. P. 91–101.

[6] V. A. Belonogov, D-blocks of characters of finite group // Amer. Math. Soc. Transl.(2). 1989. Vol. 143. P. 103–128.

[7] V. A. Belonogov, A new method for calculating p-blocks // Contemporary Math. 1995. Vol. 184, P. 49–58.

[8] V. A. Belonogov, A property of the character table for a finite group // Algebra and Logic. 2000. Vol. 39, no. 3. P. 155–159.

[9] V. A. Belonogov, A characterization of the Janko group J_1 by active fragment of its character table // Publ. Math. Debrecen. 2001. Vol. 59, no. 1-2. P. 195–202.

[10] V. A. Belonogov, Finite group with D-block of cardinality 3 // J. Math. Scien. 2010. Vol. 167, no. 6. P. 741–748.

[11] V. A. Belonogov, Small interactions in the groups $SL_3(q)$, $SU_3(q)$, $\mathrm{P}SL_3(q)$, and $PSU_3(q)$ // Trudy Inst. Mat. Mekh. UrO RAN. 1998. Vol. 5. P. 3–27 (in Russian).

[12] V. A. Belonogov, On the semiproportional character conjecture // Sib. Math. J. 2005. Vol. 46, no. 2. P. 233–245.

[13] V. A. Belonogov, On irreducible character of the groups S_n that are semiproportional on A_n or $S_n \setminus A_n$. VII // Trudy Inst. Mat. Mekh. UrO RAN. 2011. Vol. 17, no. 1. P. 3–16 (in Russian).

[14] V. A. Belonogov, On semiproportional character conjecture in groups $Sp_4(q)$ // Proc. Steklov Inst. Math. 2013. Vol. , Suppl. 1. P. S1–S18.

[15] V. A. Belonogov, On semiproportional columns of character table of groups $Sp_4(q)$ and $PSp_4(q)$ for odd q // Trudy Inst. Mat. Mekh. UrO RAN. 2015. Vol 21, no. 3, to appear (in Russian).

On Deza circulants

Sergey Goryainov
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Chelyabinsk State University, Chelyabinsk, Russia

This is joint work with Alexander Gavrilyuk and Leonid Shalaginov

A connected regular graph Γ is a Deza graph, if there exist integers a and b such that any two distinct vertices of Γ have either a or b common neighbours. A circulant is a graph that admits a cyclic group of automorphisms, i.e., it is a Cayley graph of a cyclic group.

In this talk, we report on our attempt (in progress) to classify circulants that are Deza graphs.

Finite dimensional irreducible representations of the TD-algebra

Tatsuro Ito
Anhui University, Hefei, China

In this talk, I will determine finite-dimensional irreducible representations of the tridiagonal algebra (TD-algebra).

The TD-algebra has three types: the type I includes the positive part of the quantum affine algebra $U_q(\widehat{\mathfrak{sl}}_2)$ and the q-Onsager algebra; the type II includes the Onsager algebra and its generalization; type III is related to the quantum affine algebra $U_q(\widehat{\mathfrak{sl}}_2)$ at $q = -1$. Drinfel'd polynomials play the key role in the determination of such irreducible representations.

As an application, we classify tridiagaonal pairs by explicitly constructing them as certain sort of tensor products of Leonard pairs, which will in turn provide a key tool for the classification of (P and Q)-polynomial association schemes through the investigation of their Terwilliger algebras.

On some classes of Deza graphs

Vladislav Kabanov
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Ural Federal University, Yekaterinburg, Russia

This is joint work with Leonid Shalaginov

We consider only undirected graphs, without loops and multiple edges. Let Γ be a graph. We will consider the following generalization of strongly regular graphs. Let n, k, b and a be integers such that $0 \leq a \leq b \leq k < n$. A graph Γ is a Deza graph with parameters (n, k, b, a) if

(i) Γ has exactly n vertices;

(ii) $\Gamma(u, v)$ has exactly k vertices if $u = v$, takes on one of two values b and a otherwise.

The only difference between a strongly regular graph and a Deza graph is that the size of Γ (u,v), does not necessarily depend on adjacencies. These graphs were introduced in the article by Antoine and Michel Deza [1]. So we call these graphs as Deza graphs. A strictly Deza graph is a Deza graph which is not strongly regular and has diameter 2.

Significant results for a strictly Deza graphs have got by M. Erickson, S. Fernando, W. H. Haemers, W. H. Hardy, J. Hemmiter [2].

It is easy to see the complement of a strictly Deza graph is not necessary a Deza graph and not always has the diameter 2.

We consider some class of strictly Deza graphs according to the properties of their complement graphs.

Reference

[1] A. Deza, M. Deza, The ridge graph of the metric polytope and some relatives // Polytopes: Abstract, convex and computational, ed. by T. Bisztriczky [et al.]. NATO ASI Series, Kluwer Academic, 1994. P. 359–372.

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Group factorizations, graphs and related topics

Lev Kazarin
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We study factorizations of groups in the form $G = AB$ with certain conditions on the factors A and B. The structure of the corresponding *soluble graph* gives an information on the composition factors of the group (see [1]). This approach is used also for the investigation of classes of finite groups with restriction on the normalizers of Sylow subgroups in [2]. It turns out that the Sylow graph in this paper is a subgraph of the soluble graph. Some results on the characters of groups are discussed.

Reference

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On prime graphs of finite groups

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The prime (or Gruenberg-Kegel) graph of a finite group G is an undirected simple graph whose vertex set is the set $\pi(G)$ of all prime divisors of the order of G and two vertices p and q are adjacent if and only if there exists an element of order pq in G. The prime graph of a finite group is its important arithmetical invariant, having numerous applications.

In this talk, we discuss some results on finite groups whose prime graphs have given properties.

On recent progress of 2-walk-regular graphs

Jack Koolen
University of Science and Technology of China, Hefei, China

This is based on joint work with Zhi Qiao, Alexander Gavrilyuk and Jongyook Park

t-Walk-regular graphs are a generalisation of distance-regular graphs. Many results for distance-regular graphs can be extended to 2-walk-regular graphs.

In this talk I will discuss some recent progress on 2-walk-regular graphs.

Problems on structure of finite quasifields and projective translation planes

Vladimir Levchuk
Siberian Federal University, Krasnoyarsk, Russia

This is joint work with Olga Kravtsova

Closely related problems of the construction and classification of different classes of finite non-Desargues translation planes and quasifields are being studied since in first of the last century; researches use computer calculations from 1950-th.

We introduce the orders of loop elements as a generalization of orders of group elements and similarly left and right orders. The set of orders (or left orders) of all elements of a loop is called *a spectrum* (resp., a left spectrum). For any finite proper quasifield and semifield S we study maximal subfields, their possible orders, automorphisms, *spectrums* of the loop $S^* = (S \setminus \{0\}, \circ)$ and the hypothesis: for any finite semifield S the loop S^* is one-generated.

Koolen problem for t=5

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At present the Koolen problem is solved for t at most 4.

We have reduction Koolen problem for $t = 5$ to exceptional graphs and obtain the list of parameters of exceptional graphs with the second eigenvalue 5.

On the pronormality of subgroups of odd indices in finite simple groups

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This is joint work with Anatoly Kondrat'ev and Danila Revin

A subgroup H of a group G is said to be *pronormal* in G if H and H^g are conjugate in $\langle H, H^g\rangle$ for every $g \in G$.

In [1], the following conjecture was formulated.

Conjecture. *All subgroups of odd indices are pronormal in all finite simple groups.*

In [2], the following theorem is proved.

Theorem. *All subgroups of odd indices are pronormal in the following finite simple groups*:
(1) A_n, *where* $n \geq 5$;
(2) *sporadic groups*;
(3) *groups of Lie type over fields of characteristic* 2;
(4) $L_{2^n}(q)$;
(5) $U_{2^n}(q)$;
(6) $S_{2n}(q)$, *where* $q \not\equiv \pm 3 \pmod 8$;
(7) $O_n(q)$;
(8) *exceptional groups of Lie type not isomorphic to* $E_6(q)$ *or* ${}^2E_6(q)$.

In this talk, we construct a counterexample to mentioned conjecture and discuss (in progress) a classification of finite simple groups in which all subgroups of odd indices are pronormal.

The work is supported by Russian Science Foundation (project 14-21-00065). The speaker is a winner of the competition of the Dmitry Zimin Foundation "Dynasty" for support of young mathematicians in 2013 year.

Reference

[1] E. P. Vdovin, D. O. Revin, Pronormality of Hall subgroups in finite simple groups // Sib. Math. J. 2012. Vol. 53, no. 3. P. 419–430.

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Some problems concerning vertex-symmetric graphs

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Ural Federal University, Ekaterinburg, Russia

Several problems on (mostly infinite, locally finite) vertex-symmetric graphs are formulated and discussed. By the way it is discussed what the investigation of Cayley graphs of groups gives for the investigation of general vertex-symmetric graphs.

Contributed talks

Sub-Riemannian geodesic flow for Goursat distribution

Sergey Agapov
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The following optimal control problem is considered:

$$\dot{q} = u_1 f_1(q) + u_2 f_2(q), \qquad q = (x_1, x_2, ..., x_n) \in \mathbb{R}^n, \qquad u \in \mathbb{R}^2, \tag{1}$$

where

$$f_1(q) = (1, 0, -x_2, \ldots, -x_{n-1}),$$
$$f_2(q) = (0, 1, 0, \ldots, 0)$$

are vector fields defining the distribution of two-dimensional planes in $\mathbb{R}^n$ (the so-called Goursat distribution), u is a control parameter. Boundary conditions: $q(0) = q_0$, $q(t_1) = q_1$. The functional to be minimized is as follows:

$$L = \int_0^{t_1} \sqrt{u_1^2 + u_2^2}\, dt.$$

The system (1) is completely controllable and optimal trajectories exist (see [1]). Via Pontryagin's maximum principle (see [1]) we obtain the corresponding Hamiltonian system which is proved to be completely integrable (in the Liouville sense), all the first integrals being found explicitly. The level surfaces of these integrals are described. Finally, we study motion-planning problem related to the Goursat distribution. Namely, we search for the trajectories which are periodic in some coordinates. They are related to the motion along the "prohibited" directions. This problem plays an important role in applications (for example, in robotics, see [2], [3]).

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On some class of Deza graphs without 3-cocliques

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We consider only undirected graphs, without loops and multiple edges. Let Γ be a graph. We will consider the following generalization of strongly regular graphs. Let n, k, b and a be integers such that $0 \leq a \leq b \leq k < n$. A graph Γ is a Deza graph with parameters (n, k, b, a) if

(i) Γ has exactly n vertices;

(ii) $\Gamma(u, v)$ has exactly k vertices if $u = v$, takes on one of two values b and a otherwise.

The only difference between a strongly regular graph and a Deza graph is that the size of Γ (u,v), does not necessarily depend on adjacences. These graphs were introduced in the article by Antoine and Michel Deza [1]. So we call these graphs as Deza graphs. A strictly Deza graph is a Deza graph which is not strongly regular and has diameter 2. A coedge regular Deza graph with parameter $\mu \in \{a, b\}$ is a Deza graph in which $\Gamma(u, v)$ has exactly μ vertices if $u \neq v$ and u and v are non-adjacent.

Significant results for a strictly Deza graphs have got by M. Erickson, S. Fernando, W. H. Haemers, W. H. Hardy, J. Hemmiter [2].

We consider the class of strictly Deza graphs without 3-cocliques with a small parameter a.

Theorem. Let Γ be a strictly coedge regular Deza graph without 3-cocliques and with parameters (n, k, b, a), where $\mu = a \leq 3$. Then Γ has a parameters $(10, 5, 4, 2)$ or $(8, 5, 4, 2)$. In the first case Γ is isomorphic to 2-clique extension of C_5. In the second case Γ is isomorphic to 2-clique extension of C_4.

A class of coedge regular Deza graphs with $\mu = b$ and without 3-cocliques was investigated by Galina Ermakova in [3].

Reference

[1] A. Deza, M. Deza, The ridge graph of the metric polytope and some relatives // Polytopes: Abstract, convex and computational, ed. by T. Bisztriczky [et al.]. NATO ASI Series, Kluwer Academic, 1994. P. 359–372.

[2] M. Erickson, S. Fernando, W. H. Haemers, W. H. Hardy, J. Hemmeter, Deza graphs: A generalization of strongly regular graph // J. Combin. Designs. 1999. Vol. 7, no. 6. P. 395–405.

[3] G. Ermakova, Some properties of strictly Deza graphs without 3-cocliques and with $\mu = b$// Problems of Theoretical and Applied Math.: Proc. of the 40th Conf. of Young Scientists. Yekaterinburg: IMM UB RAS. 2009. P. 19–27.

On subgraphs of graph of binary relations

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Any binary relation $R \subseteq X^2$ (where X is arbitrary set) generates on the set X^2 characteristic function: if $(x, y) \in R$, then $R(x, y) = 1$, otherwise $R(x, y) = 0$. In terms of characteristic functions on the set of all binary relations of the set X we introduced the concept of a binary reflexive relation of adjacency [1,2] and determined the algebraic system consisting of all binary relations of set and of all unordered pairs various adjacent binary relations. If X is finite set then this algebraic system is the graph ("the graph of graphs"). We investigated some its subgraphs.

The following proposition hold. Let σ and τ are adjacent relations, then 1) σ is a partial order if and only if τ is a partial order; 2) σ is a reflexive-transitive relation if and only if τ is a reflexive-transitive relation; 3) σ is an acyclic relation (acyclic digraph) if and only if τ is a acyclic relation (acyclic digraph).

We investigated some features of the structure of the graph of partial orders, the graph of reflexive-transitive relations and the graph of acyclic relations.

In particular, if X consists of n elements, and $T_0(n)$ is the number of labeled T_0-topologies defined on the set X, then the number of vertices in a graph of partial orders is $T_0(n)$, and the number of connected components is $T_0(n-1)$. Similarly in a graph of reflexive-transitive relations the number of connected components equal

$$\sum_{m=1}^{n} S(n,m)\, T_0(m-1),$$

where $S(n,m)$ is Stirling number of second kind. It is well known (see for example [3]) that the number of vertices in a graph equal

$$\sum_{m=1}^{n} S(n,m)\, T_0(m).$$

In a graph of acyclic relations the number of connected components equal

$$\sum_{p_1+\ldots+p_k=n} \frac{(-1)^{n-k}}{k} \, \frac{n!}{p_1!\ldots p_k!} \, 2^{\,(n^2-p_1^2-\ldots-p_k^2)/2}.$$

According to [4] the number of vertices in a graph equal

$$\sum_{p_1+\ldots+p_k=n} (-1)^{n-k} \frac{n!}{p_1!\ldots p_k!} \, 2^{\,(n^2-p_1^2-\ldots-p_k^2)/2}.$$

Reference

[1] Al' Dzhabri Kh.Sh.,V. I. Rodionov, The graph of partial orders // Vestn. Udmurt. Univ. Mat. Mekh. Comp. Nauki. 2013. no. 4. P. 3-12 (in Russian).

[2] Al' Dzhabri Kh.Sh., The graph of reflexive-transitive relations and the graph of finite topologies // Vestn. Udmurt. Univ. Mat. Mekh. Comp. Nauki. 2015. Issue 1. P. 3-11 (in Russian).

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Class Character Rings of groups J_1 and $O'N$

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The class character rings of group has been introduced and studied in [1].

The investigation of class character rings is the important step to the description of central units of group.

The quite exact information about class character rings has been obtained for all sporadic groups in [2]. Also there are the descriptions of unit groups of class character rings of all sporadic groups, if the class character ring is contained in some quadratic field.

Let ζ_{19} be a primitive 19th root of unity and

$$C = \zeta_{19} + \zeta_{19}^{7} + \zeta_{19}^{8} + \zeta_{19}^{11} + \zeta_{19}^{12} + \zeta_{19}^{18},$$
$$D = \zeta_{19}^{4} + \zeta_{19}^{6} + \zeta_{19}^{9} + \zeta_{19}^{10} + \zeta_{19}^{13} + \zeta_{19}^{15}.$$

By [2] the groups Janko J_1 und O'Nan $O'N$ have the following class character rings

$$K_1 = \mathbf{Z} + 77\mathbf{Z}C + 77\mathbf{Z}D \quad \text{and} \quad K_2 = \mathbf{Z} + 116963\mathbf{Z}C + 116963\mathbf{Z}D,$$

respectively.

Theorem. *The unit groups of K_1 and K_2 are*

$$\langle -1\rangle \times \langle (2+C)^{30}\rangle \times \langle (2+D)^{30}\rangle,$$
$$\langle -1\rangle \times \langle (2+C)^{1470}\rangle \times \langle (2+D)^{1470}\rangle,$$

respectively.

Reference

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[2] M. I. Molodorich, The class character rings of sporadic groups // Sib. Electron. Math. Reports. Vol. 11. P. 878–886 (in Russian).

Central Unit Group of Integral Group Ring of $GL(2,4)$

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The groups $GL(2,q)$ $(q > 2)$ have nontrivial centers. This reason is the source of certain difficulties of finding central unit groups of integral group ring of those groups. In [1] there is the complete description of central unit group of integral group ring of $GL(2,5)$.

Note that $GL(2,4) = Z(GL(2,4)) \times SL(2,4)$. So the central unit group $\mathrm{U}(Z(\mathbf{Z}SL(2,4)))$ of integral group ring $Z(\mathbf{Z}SL(2,4))$ of group $SL(2,4)(\cong A_5)$ is the subgroup of $\mathrm{U}(Z(\mathbf{Z}GL(2,4)))$. The central unit group $\mathrm{U}(Z(\mathbf{Z}A_5))$ can be found in [2].

Let β be a primitive 15th root of unity. The group $GL(2,4)$ has the character ξ of degree 3. The character field of ξ is $\mathbf{Q}(\beta+\beta^4)$. The local central unit $u_\xi(\lambda)$ can be determined for every nonzero $\lambda \in \mathbf{Q}(\beta+\beta^4)$ according to [3].

Theorem. *The central unit group* $\mathrm{U}(Z(\mathbf{Z}GL(2,4)))$ *is*

$$\langle -1\rangle \times Z(GL(2,4)) \times \mathrm{U}(Z(\mathbf{Z}SL(2,4))) \times \langle u_\xi((\beta+\beta^4)^{24})\rangle.$$

Reference

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The Parallelization of Algorithms on The Base of The Conception of Q-determinant

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We describe the approach to parallelization algorithms based on their representation as Q-determinant. The proposed approach gives the possibility of the maximal parallelization of every algorithm if it enables the parallelization.

Let $\mathcal{A}$ be an algorithm to solve the algorithmic problem $\bar{y} = F(N, B)$ where N is a parameter dimension set of the problem, B is a set of input data, $\bar{y}$ is a set of output data. Let Q be a basic set of arithmetic and logical type operations. *The expression* is called the set of operands of arithmetic or logical type that use operations from Q. Q*-term* is the map from the problem dimension to a structured set of expressions that we need to calculate one of the output variables of the problem. The set of Q-terms can be unconditional, conditional and conditionally infinite according to the structure of expression set.

Q*-determinant* is the set of Q-terms that we need to calculate each of the problem output data [1]. Let an algorithm $\mathcal{A}$ be in the form of $y_i = f_i (i = 1, \ldots, m)$ where f_i is Q-term to calculate y_i, m is the number of output data. Then we consider that the algorithm $\mathcal{A}$ represents in the form of Q-determinant.

We consider the Gauss–Jordan solution of a system of linear equations as an example of representation of the algorithm in the form of Q-determinant. Let $A\bar{x} = \bar{b}$ be a system of linear equations, where $A = [a_{ij}]_{ij=1,\ldots,n}$ is a $n \times n$ invertible matrix, $\bar{x} = (x_1, \ldots, x_n)^T$, $\bar{b} = (a_{1,n+1}, \ldots, a_{n,n+1})^T$. At the first step we suppose that the leading element is the first nonzero element of the first row of the original matrix, and at k-th step ($2 \leq k \leq n$) we select the first nonzero element of the k-th row that obtained at $(k-1)$-th step. Then the Q-determinant of Gauss–Jordan method consists of n conditional Q-terms and its representation in the form of Q-determinant has the shape

$$x_j = \left\{(u_1, w_1^j), \ldots, (u_{n!}, w_{n!}^j)\right\} \ (j = 1, \ldots, n).$$

The realization of the algorithm in the form of Q-determinant is called the process of calculating the Q-terms $f_i (i = 1, \ldots, m)$ that are included in the Q-determinant. If the calculation of all Q-terms $f_i (i = 1, \ldots, m)$ is produced at the same time and as rapid as possible, i.e. the operations of the set are executed as soon as they are ready to perform, in this case we have *the most rapid implementation of the algorithm.*

If the algorithm has some representation as flowchart then it can be represent in the form of Q-determinant [2]. The software system QStudio [3] makes possible to calculate Q-determinant of any algorithm (if the algorithm has some representation as flowchart), to find the most rapid possible implementation and to build its execution plan for the parallel system.

Reference

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Intersection of conjugated solvable subgroups in symmetric groups

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Assume that a finite group G acts on a set Ω. An element $x \in \Omega$ is called a *G-regular point* if $|xG| = |G|$, i.e. if the stabilizer of x is trivial. Define the action of the group G on Ω^k by the rule

$$g : (i_1, \dots, i_k) \mapsto (i_1 g, \dots, i_k g).$$

If G acts faithfully and transitively on Ω, then the minimal number k such that the set Ω^k contains a G-regular point is called the *base size* of G and is denoted by $b(G)$. For a positive integer m the number of G-regular orbits on Ω^m is denoted by $Reg(G, m)$ (this number equals 0 if $m < b(G)$). If H is a subgroup of G and G acts by the right multiplication on the set Ω of right cosets of H then G/H_G acts faithfully and transitively on the set Ω. (Here $H_G = \cap_{g \in G} H^g$.) In this case, we denote $b(G/H_G)$ and $Reg(G/H_G, m)$ by $b_H(G)$ and $Reg_H(G, m)$ respectively.

Thus $b_H(G)$ is the minimal number k such that there exist elements $x_1, \dots, x_k \in G$ for which

$$H^{x_1} \cap \dots \cap H^{x_k} = H_G.$$

Consider the problem 17.41 from "Kourovka notebook" [1]:

Let H be a solvable subgroup of finite group G and G does not contain nontrivial normal solvable subgroups. Are there always exist five subgroups conjugated with H such that their intersection is trivial?

The problem is reduced to the case when G is almost simple in [2]. Specifically, it is proved that if for each almost simple group G and solvable subgroup H of G condition $Reg_H(G, 5) \geq 5$ holds then for each finite nonsolvable group G and solvable subgroup H of G condition $Reg_H(G, 5) \geq 5$ holds.

We have proved the following theorem.

Theorem 1. *Let H be a solvable subgroup of an almost simple group G whose socle is isomorphic to A_n, $n \geq 5$. Then $Reg_H(G, 5) \geq 5$. In particular $b_H(G) \leq 5$.*

Reference

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On Automorphisms of Distance-Regular Graph with Intersection Array {99,84,1;1,12,99}

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We consider undirected graphs without loops or multiple edges. Given a vertex a in a graph Γ, let denote the i-neighborhood of a, i.e., the subgraph induced by Γ on the set of all its vertices that are a distance of i away from a. Let $[a] = \Gamma_1(a)$ and $a^\perp = \{a\} \cup [a]$.

If u and w are vertices separated by a distance of i in Γ, then $b_i(u,w)$ ($c_i(u,w)$) denotes the number of vertices in the intersection of $\Gamma_{i+1}(u)$ ($\Gamma_{i-1}(u)$) with $[w]$. A graph Γ of diameter d is called a distance-regular graph with an intersection array $\{b_0, b_1, \ldots, b_{d-1}; c_1, \ldots, c_d\}$ if the values $b_i(u,w)$ and $c_i(u,w)$ are independent of the choice of vertices u and w separated by a distance of i in Γ for any $i = 0, ..., d$. Let $a_i = k - b_i - c_i$. Note that, for a distance-regular graph, b_0 is the degree of the graph and $c_1 = 1$. Given a subset X of automorphisms of Γ, let $\mathrm{Fix}(X)$ denote the set of all vertices of Γ that are fixed under any automorphism from X. Let $p^l_{ij}(x,y)$ denote the number of vertices in the subgraph $\Gamma_i(x) \cap \Gamma_j(y)$ for vertices x and y separated by a distance of l in Γ. In a distance–regular graph, the numbers $p^l_{ij}(x,y)$ are independent of the choice of x and y; they are denoted by p^l_{ij} and are known as the intersection numbers of Γ.

Let $\alpha_i(g)$ denote the number of points $u \in \Gamma$ such that $d(u, u^g) = i$ for $g \in \mathrm{Aut}(\Gamma)$.

Intersection arrays distance-regular graphs, in which neighborhoods of vertices are strongly regular with parameters (99,14,1,2) were found in [1]: $\{99, 84, 1; 1, 12, 99\}$, $\{99, 84, 1; 1, 14, 99\}$, $\{99, 84, 30; 1, 6, 54\}$.

These abstracts are considered possible orders and subgraphs of fixed points hypothetical distance-regular graph with intersection array $\{99, 84, 1; 1, 12, 99\}$.

Theorem. *Let Γ be a distance-regular graph with the intersection array $\{99, 84, 1; 1, 12, 99\}$, $G = \mathrm{Aut}(\Gamma)$, g be an element of prime order p in G, and $\Omega = \mathrm{Fix}(g)$ contains at s vertices in the t antipodal classes. Then $\pi(G) \subseteq \{2, 3, 5, 7, 11\}$ and one of the following assertions holds:*

(1) *$s = 0$ and either*
  (*i*) *$p = 5$, $\alpha_1(g) = 100l$, $\alpha_2(g) = 800 - 100l$, $\alpha_3(g) = 0$, where $0 \le l \le 8$ or*
  (*ii*) *$p = 2$, $\alpha_3(g) = 16l$, $\alpha_1(g) = 16l - 40m$ for some $0 \le l \le 50$;*
(2) *$p = 11$, $t = 1$ and $\alpha_1(g) = 220l - 44$;*
(3) *$p = 7$, Ω is a t-clique and either*
  (*i*) *$t = 2$, $\alpha_3(g) = 14$, $\alpha_1(g) = 140l + 98$ or*
  (*ii*) *$t = 9$, $\alpha_3(g) = 63$, $\alpha_1(g) = 140l - 49$;*
(4) *$p = 5$, $s = 3$, $\alpha_0(g) = 3t$ and $t = 15, 20, ..., 35$;*
(5) *$p = 3$ and either*
  (*i*) *$s = 2$ and $t \in \{1, 4, ..., 25\}$ or*
  (*ii*) *$s = 5$ and $t = 1, 4, 7, ..., 22$ or*
  (*iii*) *$s = 8$ and $t = 1, 4, 7, 10, 13$;*
(6) *$p = 2$, t is even and either*
  (*i*) *$s = 2$ and $t \le 28$ or*
  (*ii*) *$s = 4$ and $t \le 28$ or*
  (*iii*) *$s = 6$ and $t \le 18$ or*
  (*iv*) *$s = 8$ and $t \le 12$.*

Corollary. *Let Γ be a distance-regular graph with the intersection array $\{99, 84, 1; 1, 12, 99\}$ and $G = \mathrm{Aut}(\Gamma)$ acts transitively on the set of vertices graph Γ. Then G is a $\{2, 3, 5\}$-group.*

The work is supported by RFBR (project No. 14-01-31298-mol_a).

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Automorphisms of strongly regular graph with parameters (1197,156,15,21)

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We consider nondirected graphs without loops and multiple edges. For vertex a of a graph Γ the subgraph $\Omega_i(a) = \{b \mid d(a,b) = i\}$ is called i-neighborhood of a in Γ. We set $[a] = \Gamma_1(a)$, $a^\perp = \{a\} \cup [a]$.

Degree of an vertex a of Γ is the number of vertices in $[a]$. Graph Γ is called regular of degree k, if the degree of any vertex is equal k. The graph Γ is called amply regular with parameters (v,k,λ,μ) if Γ is regular of degree k on v vertices, and $|[u] \cap [w]|$ is equal λ, if u adjacent to w, is equal μ, if $d(u,w) = 2$. Amply regular graph of diameter 2 is called strongly regular.

A partial geometry $pG_\alpha(s,t)$ is a geometry of points and lines such that every line has exactly $s+1$ points, every point is on $t+1$ lines (with $s > 0,\ t > 0$) and for any antiflag (P,y) there are exactly α lines z_i containing P and intersecting y. In the case $\alpha = 1$ we have generalized quadrangle $GQ(s,t)$. The incidence system $(X,\mathcal{B})$ with a point-set X and block-set $\mathcal{B}$ is called t-(V,K,Λ) *design*, if $|X| = V$, each block contains exactly K points and any t points belong to exactly Λ blocks. Every 2-design is (V,B,R,K,Λ) design, where $B = |\mathcal{B}|$, each point belong to exactly R blocks, and we have equalities $VR = BK$, $(V-1)\Lambda = R(K-1)$. Design is symmetric, if $B = V$. Design is called quasi-symmetric, if for every two blocks $B, C \in \mathcal{B}$ we have $|B \cap C| \in \{x,y\}$. Numbers x,y are called intersection numbers of quasi-symmetric design, and it is suggested that $x < y$.

Block-graph of quasi-symmetric design $(X,\mathcal{B})$ have as a vertex set $\mathcal{B}$ and two blocks $B, C \in \mathcal{B}$ are adjacent, if $|B \cap C| = y$.

Proposition 1 ([1], theorem 5.3). *Block-graph of quasi-symmetric* (V,B,R,K,Λ) *design is strongly regular with spectrum* $((R-1)K - xB + x)/(y-x)^1$, $(R-K-\Lambda+x)/(y-x)^{V-1}$, $-(K-x)/(y-x)^{B-V}$.

Derived design for t-(V,K,Λ) design $\mathcal{D} = (X,\mathcal{B})$ at $x \in X$ is design $\mathcal{D}_x$ with the point-set $X_x = X - \{x\}$ and block-set $\mathcal{B}_x = \{B - \{x\} \mid x \in B \in \mathcal{B}\}$. Design $\mathcal{E}$ is called an extension of $\mathcal{D}$,if derived design of $\mathcal{E}$ at each point is isomorphic to $\mathcal{D}$. Residual design of $\mathcal{D}$ at a block B is the design $\mathcal{D}^B$ with the point-set $X^B = X - B$ and block-set $\mathcal{B}^B = \{C \in \mathcal{B}\} \mid |B \cap C| = 0\}$.

It is known that projective plane is extendable if and only if its order is 2 or 4. P. Cameron ([1], theorem 1.35) classified extensions of symmetric 2-designs.

Proposition 2. *Let* 3-(V,K,Λ) *design* $\mathcal{E} = (X,\mathcal{B})$ *is an extension of symmetric 2-design. Then one of the following holds:*

(1) $\mathcal{E}$ *is the Hadamard* 3-$(4\Lambda+4, 2\Lambda+2, \Lambda)$ *design;*
(2) $V = (\Lambda+1)(\Lambda^2+5\Lambda+5)$ *and* $K = (\Lambda+1)(\Lambda+2)$*;*
(3) $V = 496$, $K = 40$ *and* $\Lambda = 3$.

In the case (3) we have $R = V-1 = 495$, $B = VR/K = 496 \cdot 495/40 = 6138$ and the complement to block-graph has parameters (6138,1197,156,252) and spectrum $1197^1, 9^{5642}, -105^{495}$. Hence maximal order of coclique is at most $vm/(k+m) = 6138 \cdot 105/1302 = 495$. In particular, the Hoffman bound is equal to Cvetkovich bound. The complement graph to block-graph of 3-(496,40,3) design is called Cameron monster. In [1] it is proved

Proposition 3. *For Cameron monster* Γ *the following hold:*

(1) *neighborhood of every vertex of* Γ *is strongly regular graph with parameters* $(1197, 156, 15, 21)$ *and spectrum* $156^1, 9^{741}, -15^{455}$*, and the order of coclique in this graph is at most* 105*;*

(2) *the set of blocks* C_x *containing apoint* x *of designe* $\mathcal{E}$ *is* 495*-coclique of* Γ*, for which the equality holds in Hoffman bound and Cvetkovich bound;*

(3) *subgraph* $\Gamma - C_x$ *is strongly regular graph with parameters* $(5643, 1092, 141, 228)$ *and spectrum* $1092^1, 9^{5148}, -96^{494}$*;*

(4) *for distinct points* x, y *of design* $\mathcal{E}$ *we have* $|C_x \cap C_y| = 39$*, and for coclique* $C_x - C_y$ *of graph* $\Gamma - C_y$ *the equality holds in Hoffman bound.*

In this paper automorphisms of strongly regular graph with parameters $(1197, 156, 15, 21)$ are founded.

Theorem. *Let* Γ *be a strongly regular graph with parameters* $(1197, 156, 15, 21)$*,* $G = \mathrm{Aut}(\Gamma)$*,* g *an element of prime order* p *of* G *and* $\Omega = \mathrm{Fix}(g)$*. Then* $|\Omega| \leq 171$*,* $\pi(G) \subseteq \{2, 3, 5, 7, 11, 13, 19\}$ *and one of the following holds:*

(1) Ω *is empty graph, eitrher* $p = 3$ *and* $\alpha_1(g) = 72l$*, or* $p = 7$ *and* $\alpha_1(g) = 168l - 21$*, or* $p = 19$ *and* $\alpha_1(g) = 456l + 171$*;*

(2) Ω *is* n*-clique, end either*

(*i*) $p = 13$*,* $n = 1$ *and* $\alpha_1(g) = 312l + 156$*, or*

(*ii*) $p = 2$*,* $n = 9$ *and* $\alpha_1(g) = 48l + 12$ *or* $n = 11$ *and* $\alpha_1(g) = 32l - 12$*, or*

(*iii*) $p = 5$*,* $n = 2$ *and* $\alpha_1(g) = 120l + 45$ *or* $n = 7$ *and* $\alpha_1(g) = 120l - 30$*;*

(3) Ω *is* $3t + 1$*-coclique,* $p = 3$ *and* $\alpha_1(g) = 72l + 12 - 45t$*;*

(4) Ω *contains geodesic* 2*-way and* $p \leq 13$*.*

Corollary. *Strongly regular graph with parameters* $(1197, 156, 15, 21)$ *is not vertex-symmetric.*

This work was supported by the grant of Russian Science Foundation, project no. 15-11-10025

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Faithful representations of the strong endomorphism monoid of graphs and n-uniform hypergraphs

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U. Knauer and M. Nieporte [1] proved that the monoid of strong endomorphisms of any finite undirected graph without multiple edges is isomorphic to the wreath product of a monoid with a certain small category. It was shown in [1] also that the representation fails in infinite case. In [2] we have defined a certain class of infinite undirected graphs and a certain class of infinite n-uniform hypergraphs and found faithful representations of the strong endomorphism monoid of graphs and hypergraphs from these classes. Here we generalize results of [2].

Let $G = (V, E)$ be an infinite undirected graph without multiple edges. Recall that a transformation $\varphi : V \to V$ is called a *strong endomorphism* of G if $\{x, y\} \in E \Leftrightarrow \{x\varphi, y\varphi\} \in E$ for all $x, y \in V$. The set of all strong endomorphisms of a graph G forms a monoid under composition and is denoted by $\mathrm{SEnd}\, G$. By $N(x)$ we denote the neighborhood of a vertex $x \in V$, that is, the set $\{y \in V \mid \{x, y\} \in E\}$. Let ν be the equivalence on V defined by $x\ \nu\ y \Leftrightarrow N(x) = N(y)$ for $x, y \in V$. The ν-class that contains x is denoted by x_ν. The graph G/ν with the vertex set V/ν and the edge set $\{\{a_\nu, b_\nu\} \mid \{a, b\} \in E\}$ is called the canonical strong quotient graph of the graph G.

A hypergraph is a pair $(V, \mathcal{E})$, where V is a nonempty set of elements called vertices and $\mathcal{E}$ is a family of nonempty subsets of V called edges. A hypergraph H is called an n-uniform hypergraph if it has no multiple edges and each edge consists of exactly n vertexes. By C_n we denote the class of all n-uniform hypergraphs. A transformation $\alpha : V \to V$ of a hypergraph $H \in C_n$ is called a *strong endomorphism* of the hypergraph if $A \in \mathcal{E} \Leftrightarrow A\alpha \in \mathcal{E}$ for all $A \subseteq V$, $|A| = n$. The set of all strong endomorphisms of a hypergraph H forms a monoid under composition and is denoted by $\mathrm{SEnd}\, H$.

Let $H \in C_n$ and x be a vertex of H. A neighborhood of x is defined by the formula $\mathcal{N}(x) = \{A \subseteq V : |A| = n - 1,\ \ A \cup \{x\} \in \mathcal{E}\}$. By $\rho(x)$ we denote the number of edges that contain x. For an arbitrary hypergraph $H \in C_n$ we define the equivalence relation ν on its vertex set by the rule:

$$x\ \nu\ y \Leftrightarrow \mathcal{N}(x) = \mathcal{N}(y) \text{ if } \ n \geq 2, \text{ and } x\ \nu\ y \Leftrightarrow \rho(x) = \rho(y) \text{ if } \ n \in \{0, 1\}.$$

Let H/ν be the hypergraph whose vertex set equals V/ν, and edge set consists of all $A_\nu = \{a_\nu \mid a \in A\}$, $A \subseteq V$ such that there exists a transversal T of the family A_ν with $T \in \mathcal{E}(H)$. The hypergraph H/ν is called the canonical strong quotient hypergraph of the hypergraph H.

Let $\mathrm{SEnd}_\nu\, G \subseteq \mathrm{SEnd}\, G$ ($\mathrm{SEnd}_\nu\, H \subseteq \mathrm{SEnd}\, H$) be the set of the strong endomorphisms of G (respectively H) that preserve the relation ν.

Theorem. *For an arbitrary infinite undirected graph without multiple edges G (infinite n-uniform hypergraph H) the set $\mathrm{SEnd}_\nu\, G$ ($\mathrm{SEnd}_\nu\, H$) constitutes a submonoid of $\mathrm{SEnd}\, G$ ($\mathrm{SEnd}\, H$), which is isomorphic to a wreath product of all strong injective endomorphism monoid of G/ν (H/ν) with a certain small category.*

The aforementioned results of [2] are immediate consequences of our theorem. Moreover, the theorem is true for arbitrary graphs without multiple edges and n-uniform hypergraphs.

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Vector space model using semantic relatedness

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Computers understand very little of the meaning of human language. This profoundly limits our ability to give instructions to computers, the ability of computers to explain their actions to us, and the ability of computers to analyse and to process text. Vector space models (VSM) [1] are used to overcome these limitations. However, classic VSM cannot identify semantic information [2], which results is a significantly lower expert recognition. To solve this problem, we propose a new model based on semantic relatedness (similarity) estimation. Measuring the semantic relatedness of words is a fundamental problem in natural language processing and has many useful applications, including textual entailment, word sense disambiguation, information retrieval and automatic thesaurus discovery. Experimental results indicates that the proposed model outperforms the classic VSM. All experiments are done on several linguistic resources such as dictionaries, corpuses or free encyclopedias etc.

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Complication of the state orgraph for the queuing system with distinct channels

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Let us consider a queuing system T with distinct channels $P_1, ..., P_n, n > 1$ (distinct channels are the channels with heterogeneous service efficiencies $\mu_i \neq \mu_j$ and/or separate queues which lengths are $m_k \geq 0$, $1 \leq k \leq n$). The work of such a system presupposes the presence of dispatcher device D which distributes the arrived jobs between the channels in accordance with optimization criterion L. The known problem of "slow server" shows that if the channels are distinct according to their efficiency it is important to forward the arrived job to the most efficient channel P_i, even if a lower efficiency channel P_j, $\mu_i > \mu_j$ stands idle at that moment. Another case (when each channel has a separate queue) also presupposes that in a number of cases it is more profitable to forward a job to a queue of a more efficient channel (though its queue can be filled up to a greater extent), than to place it in a shorter line of a slow channel.

In this connection a simple representation of a queuing system T with a linear state orgraph $G(T)$: $S_0 \leftrightarrow S_1 \leftrightarrow S_2 \leftrightarrow ... \leftrightarrow S_N$ of a corresponding birth and death process is impossible. Here a state's index $j \in \overline{1, N}$ is equal to the number of jobs in a system, while its maximal value is equal to the sum of line's maximal lengths and a number of serving channels: $N = n + m_1 + ... + m_n$.

$G(T)$ state orgraph in case of distinct serving channels acquires a larger number of states and ramified nonlinear form [1], [2]. This is explained by the fact that there appear the distinct variants of states $S(k_1, ..., k_n)$ with the equal sum of jobs within a system: $0 \leq k_1 + ... + k_n \leq n + m_1 + ... + m_n$. Here $k_i \in \overline{0, m_i}$ is a sum of a number of jobs within channel i and within its queue, $i \in \overline{1, n}$. Dispatcher D should take into account the occupancy rate of the serving channels and their queues $S(k_1(t), ..., k_n(t))$ at the job arrival moment t. Moreover, let us interpret t as a time-step of the system operation. Let us describe the operation of the dispatcher D of the queuing system T with distinct channels by the finite state machine K which responds to the events of job arrival to the system's input or processed job release by one of the channels. Its input alphabet is $A = \{\alpha_0, \alpha_1, ..., \alpha_n\}$ where α_0 is a signal for job arrival to the queuing system T, α_i is a signal for the finish of job processing by the channel i, $i \in \overline{1, n}$.

Depending on the occupancy rates of the channels and their queues, dispatcher D should either reject the job arrived during the time-step t or forward it to the queue of one of the channels: $D(S(k_1(t), ..., k_n(t))) = S(k_1(t+1), ..., k_n(t+1)), k_i(t) \leq k_i(t+1), i \in \overline{1, n}$ Various optimality criteria L can be used in practice: reject or idle state probability minimization, minimization of the average time of the job standing in a queue or a system, maximization of the general capacity of the system and others. Each of these criteria is not equivalent to others which leads to various dispatching protocols $D = D(L)$. Let us suggest queuing system T operation simulation with a finite state machine $K(T, L)$ [1], [2] as a method to find the optimal dispatching protocol $D = D(L)$.

As a compensation for complication of $G(T)$ graph we get a possibility of universal representation by $K(T, L)$ finite state machine in case of non-Poisson arrival, i.e. when arrival is either non-ordinary or non-stationary. Moreover, finite state machines simulating queuing systems with additional conditions, such as jobs priority, sequential compilation procedure, return of unprocessed or partially processed jobs back to the system, etc., are built up uniformly.

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Labeled graphs' vertices and edges sets clustering

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Let there be given a connected indirected loop-free graph $G(V, R)$ with vertices $\nu_i \in V$, $i \in \overline{1,n}$, $n := |V| < \infty$, and edges $r_i \in R$, $i \in \overline{1,m}$, $m := |R| < \infty$, labeled with nonnegative labels $|\nu_i| \geq 0$ and $|r_i| \geq 0$ correspondingly.

Let us consider the problem of partition of V set of vertices of G graph into disjoint clusters $U_i \subset V$, $i \in \overline{1,k}$, $k < \infty$ with fixed centers $u_i \sim U_i$, $u_i \in V$ and minimality condition for the distance between the vertex and the center of corresponding cluster. The distance $\rho^V(\nu_i, \nu_j)$ between the vertices $\nu_i, \nu_j \in V$ is defined as the minimal labels sum for the edges which constitute the path connecting these vertices. Note that introduced distance ρ^V satisfies metric separation axiom if and only if there are no edges with zero labels. Let us call conformal the introduced clustering criterion for G graph vertices and ρ^V metric.

Inverse problem is given as follows: to label the edges $r \in R$ of connected indirected loop-free finite graph $G(V, R)$ (with a given V vertexes set partition into disjoint clusters) with numeric nonnegative labels $|r| \geq 0$ generating conformal ρ^V metric. It has trivial solution: zero labels of the edges, incident with one cluster vertices, and unit labels of the rest of the edges generate conformal metric without separation axiom. The solution with separation axiom is also possible: it is enough to label the edges incident with one cluster vertices with sufficiently small labels and the rest of the edges with sufficient large ones. Accurate estimates of these labels depend on the structure of the clusters and V set of vertexes.

Thus partition $G(V, R)$ graph vertices set into clusters is equivalent to labels identification for the edges generating ρ^V conformal metric.

Transposing V set of vertices and R set of edges of $G(V, R)$ graph in the presented rule for clusters conforming we come to the similar conclusion regarding R set of edges partition into disjoint clusters: it is equivalent to labels identification for the edges generating ρ^V conformal metric.

Genuinely, we shall carry out the partition of R set of edges into disjoint clusters $W_i \subset R$, $i \in \overline{1,l}$, $l < \infty$, with fixed centers $w_i \sim W_i$, $w_i \in R$ and minimality condition for the distance between the edge and the center of corresponding cluster, according to the given labels $|\nu_i| \geq 0$ of vertices $\nu_i \in V$. The distance $\rho^R(r_i, r_j)$ between the edges $r_i, r_j \in R$ is defined as a minimal sum of the labels of the vertices on the path connecting these edges. Note that introduced distance ρ^R satisfies metric separation axiom if and only if there are no vertices with zero labels. Let us call conformal the introduced clustering criterion for G graph edges and ρ^R metric. The inverse problem is the identification of the vertices labels set generating the conformal metric ρ^R according to the given partition of R set of edges into clusters. It is solved in a similar way.

The fact that vertices clustering problem with edges labeled and edges clustering problem with vertices labeled are symmetric ones requires to develop a universal algorithm for corresponding ρ^V and ρ^R metrics calculation in vertexes and edges sets of $G(V, R)$ graph. Matrix modification of Bellman-Moore algorithm [1] which enables simple software implementation can be suggested for this purpose.

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Algorithmic recognition by spectrum

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The set of element orders of a finite group G is called *the spectrum* and denoted by $\omega(G)$, and groups with the same spectrum are said to be *isospectral*. The following question seems to be natural: if $\mathcal{M}$ is a set of positive integers, does a group G with $\omega(G) = \mathcal{M}$ exist, and if so, can one describe all such groups? We are interested in algorithmic aspect of this problem under assumption that G is simple.

Given a finite group G, the set $\mathcal{M}$ is called *almost G-spectral*, if $\mathcal{M} \subseteq \omega(G)$, $\max \mathcal{M} = \max \omega(G)$, and $\omega(H) \neq \omega(\mathcal{M})$ for every simple group H whose spectrum differs from the spectrum of G. For a finite set $\mathcal{M}$, denote by $\Omega(\mathcal{M})$ the set of all simple groups G such that $\mathcal{M}$ is almost G-spectral.

We prove the following statement.

Theorem. *Let $\mathcal{M}$ be a finite set of positive integers, $m = |\mathcal{M}|$ and $M = \max \mathcal{M}$. Then, given $\mathcal{M}$, a group G such that G lies $\Omega(\mathcal{M})$ can be determined in time polynomial in $m \log M$.*

We also are going to discuss problems of effective generation of the spectrum of a group of Lie type.

The work is supported by Russian Science Foundation (project 14-21-00065).

Locally graded groups with the minimal condition for uncomplemented subgroups

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The author has established the following theorem.

Theorem. *The locally graded group satisfies the minimal condition for uncomplemented subgroups iff it is completely factorizable or Chernikov.*

As well know, the class of locally graded groups is extremely wide.

Modules over group rings of locally finite groups with finiteness restrictions

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Let A be an $\mathbf{R}G$-module, $\mathbf{R}$ be an associative ring, G be a group. G is a finite-finitary group of automorphisms of A if $C_G(A) = 1$ and $A/C_A(g)$ is finite for any $g \in G$ [1]. Finite-finitary groups of automorphisms of A with additional restrictions were studied in [1].

Important finiteness conditions in group theory are the weak minimal condition on subgroups and the weak maximal condition on subgroups. Let G be a group, $\mathcal{M}$ be a set of subgroups of G. G is said to satisfy the weak minimal condition on $\mathcal{M}$-subgroups if for a descending series of subgroups $G_0 \geq G_1 \geq G_2 \geq \cdots \geq G_n \geq G_{n+1} \geq \cdots$, $G_n \in \mathcal{M}$, $n \in \mathbb{N}$, there exists the number $m \in \mathbb{N}$ such that an index $|G_n : G_{n+1}|$ is finite for any $n \geq m$ [2]. Similarly G is said to satisfy the weak maximal condition on $\mathcal{M}$-subgroups if for an ascending series of subgroups $G_0 \leq G_1 \leq G_2 \leq \cdots \leq G_n \leq G_{n+1} \leq \cdots$, $G_n \in \mathcal{M}$, $n \in \mathbb{N}$, there exists the number $m \in \mathbb{N}$ such that an index $|G_n : G_{n+1}|$ is finite for any $n \geq m$ [3]. These finiteness conditions were applied to investigate infinite dimensional linear periodic groups [4].

Let $\mathfrak{L}_{nf}(G)$ be the system of all subgroups H of G such that $A/C_A(H)$ is infinite. We say that G satisfies the condition W_{min-nf} if G satisfies the weak minimal condition on $\mathcal{M}$-subgroups where $\mathcal{M} = \mathfrak{L}_{nf}(G)$ and G satisfies the condition W_{max-nf} if G satisfies the weak maximal condition on $\mathcal{M}$-subgroups where $\mathcal{M} = \mathfrak{L}_{nf}(G)$.

Theorem 1. *Let A be an $\mathbf{R}G$-module, $\mathbf{R}$ be an associative ring, G be a locally finite group. If G satisfies either W_{min-nf} or W_{max-nf} then either G is a Chernikov group or G is a finite-finitary group of automorphisms of A.*

Let $G_{\mathfrak{S}}$ be the intersection of all normal subgroups K of G such that G/K is soluble.

Theorem 2. *Let A be an $\mathbf{R}G$-module, $\mathbf{R}$ be an associative ring, G be a locally soluble periodic group. If G satisfies either W_{min-nf} or W_{max-nf} then $G/G_{\mathfrak{S}}$ is soluble.*

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Decomposition of lattices of maximal antichains into the S-glued sum

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An antichain A of a poset P is called *maximal* if every element of P is comparable to an appropriate element of A. Consider the following relation on the set $MA(P)$ of all maximal antichains of P:

$$A \leqslant B \text{ iff for all } a \in A \text{ there exist } b \in B \text{ such that } a \leqslant b.$$

If P is finite, then $(MA(P), \leqslant)$ is a lattice. It is well-known (see [1]) that every finite lattice can be represented as the lattice of maximal antichains of a suitable poset. There are many such representations (see [2]), but all known representations of finite lattices as lattices of maximal antichains use posets of length 1.

In [3] V. Garg presented an application of lattices of maximal antichains in the theory of parallel computations. In his model elements of a poset were regarded as computations which are made by a single computer, and the formula $a > b$ means that the computation a starts after the computation b. Suppose that we want to minimize the number of computations without changing the lattice of maximal antichains. It is easy to show that the required poset is of maximal length. Then we obtain the following optimization problem:

Given a finite lattice L, find a finite poset P of maximal length such that $MA(P) \cong L$.

To solve this problem we use the notion of the S-glued sum (the definition can be founded in [4]). We prove the following result.

Theorem. *Let L be a finite lattice. Then the following statements are equivalent*:
(1) *L is the S-glued sum for some finite lattice S of length k.*
(2) *There exist a finite poset P of length k such that $AM(P) \cong L$.*

In our talk we also discuss an algorithm that construct the corresponding poset.

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Network project graph construction on the basis of jobs precedence table

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It is convenient to investigate project $P = \{a_1, ..., a_n\}$, consisting of a large number of connected jobs a_i, with its orgraph $\Gamma(P) = G_P(V, R)$ where project jobs $a_i \in \overline{1, n}$ are represented by its edges and the events of separate jobs start and finish coordination are represented by its vertexes $v_j \in V$, $j \in \overline{1, |V|}$. However, information about technological (logical) precedence of project jobs is used to be source information. It does not show directly a set of vertexes V. The transition from precedence table $T(P)$ to the project graph $\Gamma(P)$ involves sufficient problems, as it is often impossible to carry out such a transition without introducing additional (dummy) jobs $a_{n+1}, ..., a_{n+k}$, $k > 0$, missed in the precedence table. Evidently, we should use the minimal number ($k \to min$) of dummy jobs, as additional vertexes and edges of $G_P(V, R)$ graph make further investigation of the project P more complicated. Moreover, the specification of the minimal necessary number of additional vertexes and edges makes it possible to identify P project with its $\Gamma(P)$ orgraph unambiguously. Let us suggest the following algorithm of $\Gamma(P)$ graph construction for the project P. It consists of 5 steps [1], [2].

1. Let us pass from $T(P)$ precedence table to the $T_1(P)$ direct precedence table with only immediate predecessors for each job a_i having been left.

2. On the basis of $T_1(P)$ table let us generate $(|R|\text{x}|R|)-$ matrix $A(S_P)$ of S_P relation of direct precedence of P project jobs. In this matrix in the row corresponding to the job a_i unit elements stand for the columns of its immediate predecessors.

3. Let us correctly reindex project jobs, i.e. let us assign such indexes to them, so that job-predecessors would get an index, less than one got by the job-successor. This is always admissible if there are no logical loops in the precedence table $T(P)$ and consequently in the table $T_1(P)$. In case there exist more than one correct indexing schemes let us additionally claim that the jobs with a less number of predecessors acquire minor indexes. Let us consider $a_1, ..., a_{|R|}$ indexing correct, so all elements of $A(S_P)$ matrix within its main diagonal and over it are zeros.

4. Let us find submatrix $\left(\begin{smallmatrix} B_1 & B_2 \\ B_3 & B_4 \end{smallmatrix}\right)$ of $A(S_P)$ matrix consisting of blocks B_1, B_2, B_3, B_4 of corresponding dimensions $r\text{x}u$, $r\text{x}v$, $t\text{x}u$, $t\text{x}v$; $r, t, u, v > 0$; where B_1, B_3, B_4 blocks are filled in with unit elements, while B_2 block is filled in with zeros. If there is no such a submatrix of $(r+t)\text{x}(u+v)$, additional dummy jobs are not required and $\Gamma(P)$ orgraph is trivially constructed at a set of vertexes V. In any other case it is necessary to add at least one dummy job. For this purpose we go to the 5th step.

5. Let $A(S_P)$ matrix rows corresponding to jobs $a_{s_1}, a_{s_2}, ..., a_{s_r}$ get into B_1 and B_2 blocks, while those corresponding to $a_{s_{r+1}}, a_{s_{r+2}}, ..., a_{s_{r+t}}$ jobs get into B_3 and B_4 blocks ($r > 0$,$t > 0$). Let us add dummy job b, leading from the common start of the jobs $a_{s_1}, a_{s_2}, ..., a_{s_r}$ to common start of the jobs $a_{s_{r+1}}, a_{s_{r+2}}, ..., a_{s_{r+t}}$ to the project P and get back to step 1.

Let us repeat steps 1-5, until project P is filled up with all the necessary dummy jobs. After this graphical realization of the project orgraph $\Gamma(P)$ becomes trivial. As step 4 to 5 transitional condition is a NC for adding a dummy job, the algorithm guarantees the minimal number of such additions.

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On the decomposition of elementary transvection in elementary group

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We consider the following data: an elementary net $\sigma = (\sigma_{ij})$ (elementary carpet) of the additive subgroups of a commutative ring (the net without the diagonal) of the order n, a derived net $\omega = (\omega_{ij})$, which depends of the net σ, the net $\Omega = (\Omega_{ij})$, which associated with the elementary group $E(\sigma)$, where $\omega \subseteq \sigma \subseteq \Omega$ and the net Ω is the least (complemented) net among the all nets which contain the elementary net σ. We prove that every elementary transvection $t_{ij}(\alpha)$ can be decomposed as a product of two matrixes M_1 and M_2, where M_1 is the element of the group $\langle t_{ij}(\sigma_{ij}), t_{ji}(\sigma_{ji})\rangle$, M_2 is the element of the net group $G(\tau)$ and the net τ has the representation $\tau = \begin{pmatrix} \Omega_{11} & \omega_{12} \\ \omega_{21} & \Omega_{22} \end{pmatrix}$.

The work of V. A. Koibaev was supported by the RFBR (project 13-01-00469). The results of the present paper were obtained in the frame of the state assignment of the Russian Ministry of Education.

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Automorphisms of a distance-regular graph with intersection array $\{100, 66, 1; 1, 33, 100\}$

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A. A. Makhnev and D. V. Paduchikh have found [1] intersection arrays of distance-regular graphs, in which neighborhoods of vertices are strongly-regular graphs with second eigenvalue 3. A. A. Makhnev suggested the program to research of automorphisms of these distance-regular graphs. In this moment cases $\{100, 66, 1; 1, 33, 100\}$, $\{176, 150, 1; 1, 25, 176\}$ and $\{256, 204, 1; 1, 51, 256\}$ are not investigated.

In this paper are researching possible orders and subgraphs of fixed points of automorphisms of a hypothetical distance-regular graph with intersection array $\{100, 66, 1; 1, 33, 100\}$. Possible automorphisms of a strongly-regular graph with parameters (100,33,8,12) found in [2].

Theorem 1. *Let Γ be a distance-regular graph with intersection array $\{100, 66, 1; 1, 33, 100\}$, $G = \mathrm{Aut}(\Gamma)$, g be an element of G with prime order p and $\Omega = \mathrm{Fix}(g)$ contains along s vertices in t antipodal classes. Then $\pi(G) \subseteq \{2, 3, 5, 7, 11, 29, 31, 101\}$ and one of the following assertions holds:*

(1) *Ω is an empty graph and either $p = 101$, $\alpha_1(g) = 101$, or $p = 3$, $\alpha_1(g) = 60m + 27l + 21$;*

(2) *$p = 31$, Ω is a distance-regular graph with intersection array $\{7, 4, 1; 1, 2, 7\}$;*

(3) *$p = 29$, Ω is a distance-regular graph with intersection array $\{13, 8, 1; 1, 4, 13\}$;*

(4) *$p = 11$ and $t = 2, 13, 24$;*

(5) *$p = 7$ and either Ω is a distance-regular graph with intersection array $\{16, 10, 1; 1, 5, 16\}$, or $t = 24, 31$;*

(6) *$p = 5$ and $t = 1, 16, 21, 26, 31$;*

(7) *$p = 3$, $s = 3$ and $t = 2, 5, ..., 32$;*

(8) *$p = 2$, t is odd and either $s = 3$, $t = 1, 3, 5, ..., 33$, or $s = 1$ and $t = 1, 3, 5, ..., 11$.*

Theorem 2. *Let Γ be a distance-regular graph with intersection array $\{100, 66, 1; 1, 33, 100\}$, in which neighbourhoods of vertices are strongly-regular graphs with parameters $(100, 33, 8, 12)$, $G = \mathrm{Aut}(\Gamma)$, g be an element of G with prime order $p > 2$ and $\Omega = \mathrm{Fix}(g)$ is not empty graph, which contains along s vertices in t antipodal classes. Then $\pi(G) \subseteq \{2, 3, 11, 101\}$ and one of the following assertions holds:*

(1) *$p = 11$, $s = 3$ and $t = 2$;*

(2) *$p = 3$, $s = 3$ and either $t = 5$, Ω is an union of isolated 5-cliques, or $t = 5, 8, ..., 17$ and neighbourhoods of vertices in Ω are cocliques, or $t = 11, 14, ..., 26$ and neighbourhood of any vertex in Ω contains geodesic 2-path;*

(3) *$p = 2$, either Ω contained in antipodal class, or $t = 5$ and Ω is an union of isolated 5-cliques and $s = 1, 3$, or neighbourhoods of vertices in Ω are unions of isolated cliques and $s = 3$, $t = 3, 5$, or neighbourhood of any vertex in Ω contains geodesic 2-path and $s = 3$, $t = 7, 9, ..., 33$.*

Corollary. *A distance-regular graph with intersection array $\{100, 66, 1; 1, 33, 100\}$ is not vertex-transitive.*

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Algebra variety properties given by identities of derived objects

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We consider varieties of associative algebras over a field or over $\mathbb{Z}$, i.e. varieties of associative rings.

With any algebra $\langle A, +, \cdot \rangle$, two semigroups and a Lie algebra are associated in a natural way. The first semigroup is just the multiplicative semigroup $\langle A, \cdot \rangle$ of the algebra. The second one is so-called *adjoint* semigroup $\langle A, \circ \rangle$, where the multiplication $\circ$ (sometimes referred to as *circle composition*) is defined by letting $a \circ b = a + b - ab$ for all $a, b \in A$. The Lie algebra is the algebra $\langle A, +, [,] \rangle$, where $[x, y] = x \cdot y - y \cdot x$.

In this talk we discuss some algebra variety properties which are given by identities of these semigroups or by identities of this Lie algebra.

Artificial Landmark Placement for Mobile Robot Navigation

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The integration of robots into real world environments is a difficult problem. The problem of improving the performance of robots by autonomously adapting them to different tasks and environments has been extensively studied recently (see e.g. [1,2]). However, the problem is still far from being solved. Therefore, for many applications, it is preferable to use different adaptations of environments. In particular, artificial visual landmark navigation has been widely studied (see e.g. [3,4]). We consider the problem of artificial visual landmark placement for a low-cost mobile robot navigation. We assume that an instrumentation of the environment with artificial visual landmarks is used to improve the mobile robot performance. We use a humanoid robot for the placement of artificial visual landmarks. We need to minimize the path length of the humanoid robot.

We assume that the mobile robot must solve some task. To solve the task, the robot must visit the multiset of points $U = \{U_1, U_2, \dots, U_n\}$ in the predetermined sequence $U_1, U_2, \dots, U_n$. Let V be the set such that $V = \{V_i \mid V_i \in U\}$. Let $S = \{S_1, S_2, \dots, S_k\} \subseteq U$ be the set of all points that must be equipped with artificial visual landmarks. Let G be the weighted complete graph with the set of vertices V and the weight function F. We assume that the robots have the same speed. It is assumed that the robots have arbitrary initial positions. Also, we assume that the robots do not stop after the start of movement. Let $t_M(S_i)$ be the time such that the mobile robot is located at S_i for the first time, $i \in \{1, 2, \dots, k\}$. Let $t_H(S_i)$ be the time such that the humanoid robot is located at S_i, $i \in \{1, 2, \dots, k\}$. Let A be the weighted complete graph with the set of vertices S and the weight function $F|_S$.

Artificial Visual Landmark Placement for Mobile Robot Navigation (LP):

Instance: *Weighted complete graphs G and A, the sequence $U_1, U_2, \dots, U_n$.*

Task: *Find the shortest tour of S such that $t_M(S_i) \leq t_H(S_i)$, $i \in \{1, 2, \dots, k\}$?*

The decision version of LP can be formulated as following.

LP_D:

Instance: *Weighted complete graphs G and A, the sequence $U_1, U_2, \dots, U_n$, positive integer t.*

Task: *Is there a tour of S such that $t_M(S_i) \leq t_H(S_i) \leq t$, $i \in \{1, 2, \dots, k\}$?*

Theorem 1. LP_D *is* **NP***-complete.*

Theorem 2. LP *is* $\mathbf{FP}^{\mathbf{NP}}$*-complete.*

In view of intractability of LP, we propose an explicit reduction from LP_D to the satisfiability problem.

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Chain varieties of monoids

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Since the first half of 1960's, more than 200 articles appeared where the lattice of semigroup varieties is investigated. Many deep and interesting results were obtained here (see the survey articles [1, 1]). In contrast, only a few isolated facts is known so far about the lattice of monoid varieties. We know only two works devoted to examination of this lattice, namely [3, 4].

One of the first natural steps in investigation of varietal lattice of algebras of any type is a description of varieties whose lattice of subvarieties is a chain. Varieties with such a property are called *chain* varieties. Non-group chain varieties of semigroups and locally finite chain varieties of groups have been completely determined in [5] and [6] respectively. The problem of a complete description of chain varieties of groups seems to be extremely difficult. To confirm this claim, we refer to the fact that there exist uncountably many periodic group varieties whose subvariety lattice is the 3-element chain [7].

We completely classify all non-group chain varieties of monoids. The description is given in a language of identities and in terms of minimal forbidden subvarieties. We do not reproduce the description here because the corresponding list of varieties is quite lengthy. It consists of two countable series of varieties and 28 "sporadic" varieties. It is interesting to note that one of these two countable series of varieties appeared recently in the article [8] in connection with an investigation of so-called Cross varieties of monoids.

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A graph clustering problem with bounded number of clusters

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We consider a version of the graph clustering problem, so called correlation clustering or graph approximation problem which is one of most visual formalizations of the clustering problem. The objective of the clustering problem is to partition of objects (data elements) into a family of subsets (i.e., clusters) such that objects within a cluster are more similar to one another than objects in different clusters. In the graph approximation problem one has to partition the vertices of a graph into clusters taking into consideration the edge structure of the graph: the goal is to minimize the number of edges between the clusters and the number of missing edges within the clusters. For statements and various interpretations of this problem, see [1–4].

We consider only *simple* graphs, i.e., the graphs without loops and multiple edges. A graph is called a *cluster graph* if each of its connected components is a complete graph. Denote by $\mathcal{M}_k(V)$ the set of all cluster graphs on a vertex set V consisting of exactly k nonempty connected components, $2 \le k \le |V|$. If $G_1 = (V, E_1)$ and $G_2 = (V, E_2)$ are graphs on the same vertex set V, then the *distance* between them is defined as $\rho(G_1, G_2) = |E_1 \setminus E_2| + |E_2 \setminus E_1|$.

The following version of the graph clustering problem is known as the *graph approximation problem* or *correlation clustering.*

Problem $\mathbf{A_k}$. *Given a graph $G = (V, E)$ and an integer k, $2 \le k \le |V|$, find a graph $M^* \in \mathcal{M}_k(V)$ such that*

$$\rho(G, M^*) = \min_{M \in \mathcal{M}_k(V)} \rho(G, M). \tag{1}$$

In machine learning clustering methods fall under the section of *unsupervised learning.* At the same time *semi-supervised* clustering methods use limited supervision. For example, relatively few objects are labeled (i.e., are assigned to clusters), whereas a large number of objects are unlabeled. This leads to the following version of the graph clustering problem.

Problem $\mathbf{A_k^+}$. *Given a graph $G = (V, E)$, an integer k, $2 \le k \le |V|$, and a set $X = \{x_1, \dots, x_k\} \subset V$ ($x_i \ne x_j$ unless $i = j$), find a graph $M^* \in \mathcal{M}_k(V)$ provided that minimum in (1) is taken over all cluster graphs $M \in \mathcal{M}_k(V)$ such that $x_i \in V_i$, $i = 1, \dots, k$, where V_i is the vertex set of ith cluster (connected component) of the graph M.*

Problem $\mathbf{A_k}$ is known to be *NP*-hard for any fixed integer $k \geqslant 2$ [3]. We prove that problem $\mathbf{A_k^+}$ is *NP*-hard for any fixed integer $k \geqslant 2$, and for $k = 2, 3$ we propose constant-factor approximation polynomial-time algorithms for problems $\mathbf{A_k}$ and $\mathbf{A_k^+}$.

Research of the first author was supported by RSF grant 15-11-10009.

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Automorphisms of graph with intersection array $\{169, 126, 1; 1, 42, 169\}$

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We consider nondirected graphs without loops and multiple edges. For vertex a of a graph Γ the subgraph $\Omega_i(a) = \{b \mid d(a,b) = i\}$ is called i-neighborhood of a in Γ. We set $[a] = \Gamma_1(a)$, $a^\perp = \{a\} \cup [a]$.

Degree of an vertex a of Γ is the number of vertices in $[a]$. Graph Γ is called regular of degree k, if the degree of any vertex is equal k. The graph Γ is called amply regular with parameters (v, k, λ, μ) if Γ is regular of degree k on v vertices, and $|[u] \cap [w]|$ is equal λ, if u adjacent to w, is equal μ, if $d(u,w) = 2$. Amply regular graph of diameter 2 is called strongly regular.

Jack Koolen suggested the problem investigation of distance-regular graphs whose local subgraphs are strongly regular graphs with the second eigenvalue at most t for some natural number t. For $t = 3$ A. Kagazezheva and A. Makhnev [1] proved the next result

Proposition. *Let Γ be a distance-regular graph with strongly regular local subgraphs having eigenvalue 3 and parameters $(v', k', 5, \mu')$. Then local subgraphs either isomorphic triangular graph $T(7)$ and Γ is a half graph of 7-cube, or have parameters $(169, 42, 5, 12)$ and Γ has intersection array $\{169, 126, 1; 1, 42, 169\}$.*

In this paper it is founded automorphisms of distance-regular graph with intersection array $\{169, 126, 1; 1, 42, 169\}$.

Theorem. *Let Γ be a distance-regular graph with intersection array $\{169, 126, 1; 1, 42, 169\}$, and local subgraphs of Γ are strongly regular with parameters $(169, 42, 5, 12)$, $G = \mathrm{Aut}(\Gamma)$, g — an element of G prime order $p > 2$ and $\Omega = \mathrm{Fix}(g)$ is nonempty graph containing s vertices in t antipodal classes. Then $\pi(G) \subseteq \{2, 3, 5, 7, 13, 17\}$ and one of the following holds:*

(1) some $\langle g\rangle$-orbit on $\Gamma - \Omega$ contains geodesic 2-way, either $p = 7$ and $t = 2$, or $p = 5$ and Ω is a distance-regular graph with intersection array $\{9, 6, 1; 1, 2, 9\}$;

(2) some $\langle g\rangle$-orbit on $\Gamma - \Omega$ is clique, $p = 3$ and either $s = 4$, $t = 2, 5$ and Ω is the union of 4 isolated t-cliques, or $s = 1$ and Ω is 2-clique;

(3) every $\langle g\rangle$-orbit on $\Gamma - \Omega$ is coclique, either $p = 13$, Ω is an antipodal class, or $p = 5$ and $t = 40$, or $p = 3$, $s = 4$ and $t = 14$.

Corollary. *Let Γ be a distance-regular graph with intersection array $\{169, 126, 1; 1, 42, 169\}$, and local subgraphs of Γ are strongly regular with parameters $(169, 42, 5, 12)$. If $G = \mathrm{Aut}(\Gamma)$ is nonsolvable group acting transitively on the vertex set of Γ, then $S = S(G)$ is an elementary abelian 2-group, $\bar{G} = G/S$ is isomorphic to $Sp_4(4)$, for any vertex $a \in \Gamma$ we have $G_a = 2^6 : (Z_3 \times A_5)$, S contains normal in G subgroup K of order 4, regular on each antipodal class, $|S : S_{\{F\}}| = 2$ for antipodal class F, S/K is irreducible $F_2Sp_4(4)$-module of order $2^8, 2^{16}, 2^{32}$ and $C_S(f) = K$ for every element f of order 17 inG.*

This work was supported by the grant of Russian Science Foundation, project no. 14-11-00061.

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Some combinatorial problems in symmetric groups

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One of the actual problems in the group theory is a representation of an element of the group by the word of generators. For example, the study of infinite groups saturated by a set of finite groups is this one. Also these tasks are arised in the many practical problems. For example, in the design of the topology of a multiprocessor computing system (MCS). In this case the model of MCS will be presented as the Cayley graph in which the the processors are the vertices of the graph and the edges correspond to physical connections between processors.

Let S_n be the symmetric group of degree n and $x = (1, 2)$, $y = (1, 2, \ldots, n)$ be generators of S_n. Let $x < y$ and elements of S_n written by words of generators be lenlex ordered. $\pi_i(n)$ denote elements of S_n which have the maximal length in this ordering. Our hypothesis for $n \geq 6$ is following.

1. For even n there is the only one permutation $\pi(n)$:

$$\pi(n) = (1, 3, n, 2) \prod_{i=1}^{\frac{n-4}{2}} (a_i, b_i), \quad a_i = 3 + i, \quad b_i = n - i.$$

2. For odd n there are the only two permutations $\pi_1(n)$ and $\pi_2(n)$:

$$\pi_1(n) = (1, 2)(a_1, b_1, a_2, b_2, \ldots, \frac{n+3}{2}), \quad a_i = 2 + i, \quad b_i = n + 1 - i, \quad i \leq \frac{n-3}{2};$$

$$\pi_2(n) = (1, 2)(3, n, \frac{n+3}{2}, a_1, b_1, a_2, b_2, \ldots), \quad a_i = \frac{n+3}{2} - i, \quad b_i = \frac{n+3}{2} + i, \quad i \leq \frac{n-5}{2}.$$

The following table shows examples of $\pi_i(n)$ for $6 \leq n \leq 12$ which are obtained by computer computations.

Group	Permutations $\pi_i(n)$	Product of cycles $\pi_i(n)$
S_6	$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 6 & 5 & 4 & 2 \end{pmatrix}$	(1,3,6,2)(4,5)
S_7	$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 1 & 7 & 6 & 3 & 5 & 4 \end{pmatrix}$	(1,2)(3,7,4,6,5)
	$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 1 & 7 & 6 & 4 & 3 & 5 \end{pmatrix}$	(1,2)(3,7,5,4,6)
S_8	$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 1 & 8 & 7 & 6 & 5 & 4 & 2 \end{pmatrix}$	(1,3,8,2)(4,7)(5,6)
S_9	$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 1 & 9 & 8 & 7 & 3 & 6 & 5 & 4 \end{pmatrix}$	(1,2)(3,9,4,8,5,7,6)
	$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 1 & 9 & 8 & 7 & 5 & 4 & 3 & 6 \end{pmatrix}$	(1,2)(3,9,6,5,7,4,8)
S_{10}	$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 3 & 1 & 10 & 9 & 8 & 7 & 6 & 5 & 4 & 2 \end{pmatrix}$	(1,3,10,2)(4,9)(5,8)(6,7)
S_{11}	$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 2 & 1 & 11 & 10 & 9 & 8 & 3 & 7 & 6 & 5 & 4 \end{pmatrix}$	(1,2)(3,11,4,10,5,9,6,8,7)
	$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 2 & 1 & 11 & 10 & 9 & 8 & 6 & 5 & 4 & 3 & 7 \end{pmatrix}$	(1,2)(3,11,7,6,8,5,9,4,10)
S_{12}	$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 3 & 1 & 12 & 11 & 10 & 9 & 8 & 7 & 6 & 5 & 4 & 2 \end{pmatrix}$	(1,3,12,2)(4,11)(5,10)(6,9)(7,8)

The work is supported by the Grant of the President of the Russian Federation (Project MD-3952.2015.9).

Finite groups with large irreducible character

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Let G be a finite group and Θ be an ordinary irreducible character of G. We study finite groups having an ordinary irreducible character Θ such that $|G| \leq 2\Theta(1)^2$. Groups with a character of large degree were investigated by N. Snyder in [1].

Theorem *Let G be a finite group with an ordinary irreducible character Θ such that $\Theta(1) = pq$, where p and q are different primes. If $2\Theta(1)^2 \geq |G|$, then G has an abelian normal subgroup of index pq.*

Note that some sporadic simple groups G have an irreducible character Θ such that $|G| < 3\Theta(1)^2$. For instance, the sporadic group $Th = F_{3|3}$ of Thompson has an irreducible character Θ of degree $\Theta(1) = 190373976$, so that $|Th| < 2,51\Theta(1)^2$.

It is easy to see that the Frobenius group of order $n(n+1)$ with $n = pq$ and $n+1 = 2^m$ is an example of the group with an irreducible character Θ with degree pq.

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Automorphisms of local subgraphs of pseudogeometric graph for $pG_3(7,75)$

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We consider nondirected graphs without loops and multiple edges. For vertex a of a graph Γ the subgraph $\Omega_i(a) = \{b \mid d(a,b) = i\}$ is called i-neighborhood of a in Γ. We set $[a] = \Gamma_1(a)$, $a^\perp = \{a\} \cup [a]$. For a vertex subset S of a graph Γ we denote as $\Gamma(S)$ the set $\cap_{a\in S}([a] - S)$.

Degree of an vertex a of Γ is the number of vertices in $[a]$. Graph Γ is called regular of degree k, if the degree of any vertex is equal k. The graph Γ is called amply regular with parameters (v, k, λ, μ) if Γ is regular of degree k on v vertices, and $|[u] \cap [w]|$ is equal λ, if u adjacent to w, is equal μ, if $d(u, w) = 2$. Amply regular graph of diameter 2 is called strongly regular.

By $K_{m\times n}$ we denote the complete bipartite graph with m parties of order n. Graph on the set $X \times Y$ is called $p \times q$*-grid*, if $|X| = p$, $|Y| = q$, and pairs (x_1, y_1) and (x_2, y_2) are adjacent if and only if $x_1 = x_2$ or $y_1 = y_2$. By mK_n we denote the union of m isolated n-cliques.

A partial geometry $pG_\alpha(s, t)$ is a geometry of points and lines such that every line has exactly $s + 1$ points, every point is on $t + 1$ lines (with $s > 0,\ t > 0$) and for any antiflag (P, y) there are exactly α lines z_i containing P and intersecting y. In the case $\alpha = 1$ we have generalized quadrangle $GQ(s, t)$.

Point-graph of a geometry (P, L) of points and lines has P as a vertex set, and two vertices a, b are adjacent if a, b belong to some line. Point-graph of partial geometry $pG_\alpha(s, t)$ is strongly regular with parameters $v = (s+1)(1+st/\alpha)$, $k = s(t+1)$, $\lambda = (s-1)+(\alpha-1)t$, $\mu = \alpha(t+1)$. Strongly regular graph with this parameters for some natural numbers α, s, t is called pseudogeometric graph for $pG_\alpha(s, t)$.

A graph Γ is called t-izoregular, if for every $i \le t$ and for every i-vertex subset S the number $|\Gamma(S)|$ is depend only from isomorphic type of the subgraph induced by S. A graph on v vertices is called absolute izoregular, if it is $(v - 1)$-izoregular. Finally t-izoregular graph Γ is called exactly t-izoregular, if it is not $(t + 1)$-izoregular. Cameron [1] proved that every 5-izoregular graph Γ is absolute izoregular and is isomorphic pentagon, 3×3-grid, complete multipartite graph $K_{n\times m}$ or its complement. Further every exactly 4-izoregular graph is pseudogeometric for $pG_r(2r, 2r^3 + 3r^2 - 1)$ or its complement. Let $Izo(r)$ be a pseudogeometric graph for $pG_r(2r, 2r^3 + 3r^2 - 1)$. For $r = 1$ we have the point graph of $GQ(2, 4)$, and for $r = 2$ we have MacLaughlin graph.

For every vertex a of a graph $Izo(r)$ the subgraph $\Gamma(a)$ is pseudogeometric for $pG_{r-1}(2r-1, r^3+r^2 - r - 1)$. Makhnev [1] proved that pseudogeometric graph for $pG_{r-1}(2r - 1, r^3 + r^2 - r - 1)$ does not exist for $r = 3$. Automorphisms of 2-neighborhood Σ of some vertex of $Izo(3)$ and local subgraphs of Σ were determined by M. Nirova, M. Isakova and A. Tokbaeva [3], [4], [5].

Graph $Izo(4)$ has parameters $(3159, 1408, 532, 704)$ and for any vertex a subgraph $\Sigma = [a]$ is pseudogeometric for $pG_3(7, 75)$ and has parameters $(1408, 532, 156, 228)$. Further, for any vertex $b \in \Sigma$ subgraph $\Delta = \Sigma(b)$ is pseudogeometric for $pG_2(6, 25)$ and has parameters $(532, 156, 30, 52)$, subgraph $\Delta' = \Sigma_2(b)$ is strongly regular with parameters $(875, 304, 78, 120)$. In this paper automorphisms of strongly regular with parameters $(532, 156, 30, 52)$ are determined.

Theorem. *Let Γ be a strongly regular with parameters $(532, 156, 30, 52)$, $G = \mathrm{Aut}(\Gamma)$, g is an element of prime order p of G and $\Omega = \mathrm{Fix}(g)$. Then $\pi(G) \subseteq \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$ and one of the following holds:*

(1) Ω is empty graph, either $p = 19$ and$\alpha_1(g) = 152$, or $p = 7$ and $\alpha_1(g) = 210l - 28$, or $p = 2$ and $\alpha_1(g) = 30l + 16$;

(2) Ω is n-clique, either $p = 3$, $n = 1$ and $\alpha_1(g) = 90l + 36$, or $p = 5$, $n = 2$ and $\alpha_1(g) = 150l - 20$ or $n = 7$ and $\alpha_1(g) = 150l - 30$;

(3) Ω *is* l*-coclique, either* $p = 2$ *and* $\alpha_1(g) = 4m + 152 - 60l$*, or* $p = 13$ *and* $\alpha_1(g) = 13(4s - 30t - 14)$*, where* $m = 13s - 1$*;*

(4) Ω *contains geodesic* 2*-way and* $p \leq 23$*,* $p \neq 19$*.*

This work was supported by the grant of Russian Science Foundation, project no. 15-11-10025.

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Spectral properties of the Star graph

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Spectral properties of Cayley graphs on the symmetric group Sym_n generated by transpositions have studied intensively last years. In 2000 it was shown by J. Friedman [1] that the Cayley graph on Sym_n with respect to a set of $n-1$ transpositions has the smallest non-zero eigenvalue $\lambda_2 \leqslant 1$, with equality iff for some i we have $T = \{(i,j)|j \neq i\}$. The multiplicity of this eigenvalue is

$$mul(\lambda_2) \geqslant n-1. \tag{1}$$

For example, if $T = \{(1,2),(2,3),\ldots,(n-1,n)\}$ then we have the Bubble-sort graph whose spectral properties were investigated by R. Bacher in [2].

In this paper we study spectral properties of the *Star graph* S_n that is the Cayley graph on Sym_n with the generating set $T = \{(1,2),(1,3),\ldots,(1,n)\}$. In 2009 A. Abdollahi and E. Vatandoost conjectured [3] that the spectrum of S_n is integral, moreover it contains all integers in the range from $-(n-1)$ up to $n-1$ (with the sole exception that when $n \leqslant 3$, zero is not an eigenvalue of S_n). This conjecture was proved by R. Krakovski and B. Mohar [4] in 2012.

We investigate multiplicity of eigenvalues of the Star graph S_n. Using the standard representation theory [5] their exact values were found for $4 \leqslant n \leqslant 13$. The obtained data show an oscillating distribution of eigenvalue multiplicities. One can assume that this behavior of multiplicities will be also kept for large n. Let us note that typically the distribution of eigenvalue multiplicities for known distance-regular graphs is unimodal. However, the Star graph is not distance-regular. It is also shown that the low bound (1) for $mul(\lambda_2)$ is achieved only for $2 \leqslant n \leqslant 5$ in S_n. The following result is given.

Theorem. *The values* $\pm(n-2)$ *are eigenvalues of* S_n *with multiplicity* $(n-2)(n-1)$.

Most of the talk is based on results from [6]. The work has been supported by RFBS Grant 15-01-05867 and Grant NSh-1939.2014.1 of President of Russia for Leading Scientific Schools.

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On finite 5-primary groups G with disconnected Gruenberg — Kegel graph

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Let G be a finite group. Denote by $\pi(G)$ the set of all prime divisors of the order of G. *Prime graph* (or *Gruenberg — Kegel graph*) $\Gamma(G)$ of G is defined as the graph with vertex set $\pi(G)$, in which two distinct vertices p and q are adjacent if and only if G contains an element of order pq. A group G is called *n-primary* if $|\pi(G)| = n$. We denote the number of connected components of $\Gamma(G)$ by $s(G)$, and the set of its connected components by $\{\pi_i(G) \mid 1 \leq i \leq s(G)\}$; for the group G of even order believe that $2 \in \pi_1(G)$.

Kondrat'ev determined finite almost simple 5-primary groups and their Gruenberg — Kegel graphs [1]. The author together with A. S. Kondrat'ev [2] obtained a description of chief factors of the commutator subgroups of finite non-solvable 5-primary groups G with disconnected Gruenberg–Kegel graph in the case when $G/F(G)$ is almost simple n-primary group for $n \leq 4$. Our aim is to describe 5-primary groups G with disconnected prime graph in the remaining cases. It is natural to begin the study by imposing certain restrictions on the component $\pi_1(G)$. The result of this work is describing 5-primary groups G with disconnected prime graph such that either $\pi_1(G) = \{2\}$, or $3 \notin \pi_1(G) \neq \{2\}$ and $3 \in \pi(G)$.

We prove the following two theorems. Each of the items of these theorems is realizing.

Theorem 1. *Let G be a finite 5-primary group and $\pi_1(G) = \{2\}$. Then one of the following conditions holds:*

(1) *$G \cong O(G) \leftthreetimes S$ is Frobenius group, where $O(G)$ is 4-primary abelian group and S is cyclic 2-group or generalized quaternion group;*

(2) *G is Frobenius group with kernel $O_2(G)$ and 4-primary complement;*

(3) *$G \cong A \leftthreetimes (B \leftthreetimes C)$ is 2-frobenius group, where $A = O_2(G)$, B is cyclic 4-primary $2'$-group and C is cyclic 2-group;*

(4) *$G \cong L_2(r)$, $r \geq 65537$ is Mersenne or Ferma prime and $|\pi(r^2 - 1)| = 4$;*

(5) *$\overline{G} = G/O_2(G) \cong L_2(2^m)$, where either $m \in \{6, 8, 9\}$, or $m \geq 11$ is prime. If $O_2(G) \neq 1$, then $O_2(G)$ is a direct product of minimal normal subgroups of order 2^{2m} from G, each of these as $\overline{G}$-module is isomorphic to the natural $GF(2^m)SL_2(2^m)$-module;*

(6) *$\overline{G} = G/O_2(G) \cong Sz(q)$, where $q = 2^p$, $p \geq 7$ and $q - 1$ primes, $|\pi(q - \varepsilon\sqrt{2q} + 1)| = 2$ and $|\pi(q + \varepsilon\sqrt{2q} + 1)| = 1$ for $\varepsilon \in \{+, -\}$, $5 \in \pi(q - \varepsilon\sqrt{2q} + 1)$. If $O_2(G) \neq 1$, then $O_2(G)$ is a direct product of minimal normal subgroups of order q^4 from G, each of these as $\overline{G}$-module is isomorphic to the natural $GF(q)Sz(q)$-module of dimension 4.*

Theorem 2. *Let G be a finite 5-primary group with disconnected prime graph, $\overline{G} = G/F(G)$ is almost simple 5-primary group, $3 \in \pi(G)$ and $3 \notin \pi_1(G) \neq \{2\}$. Then one of the following conditions holds:*

(1) *G is isomorphic to $L_2(5^3)$ or $L_2(17^3)$;*

(2) *$G \cong L_2(p)$, where either $p \geq 65537$ is Mersenne or Ferma prime and $|\pi(p^2 - 1)| = 4$, or $p \geq 41$ is prime, $|\pi(p^2 - 1)| = 4$ and $3 \in \pi(\frac{p+1}{2})$;*

(3) *G is isomorphic to $L_2(3^r)$ or $PGL_2(3^r)$, where r is odd prime, $|\pi(3^{2r} - 1)| = 4$ and $r \notin \pi(G)$;*

(4) *$G \cong L_2(p^r)$, where $p \in \{5, 17\}$, r is odd prime, $|\pi(p^{2r} - 1)| = 4$, $3 \in \pi(\frac{p^r+1}{2})$ and $r \notin \pi(G)$.*

The work was supported by the Russian Scientific Foundation (project No. 14-11-00061).

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On adjacency for the prime graph of a finite simple group

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The prime (or Gruenberg-Kegel) graph $\Gamma(G)$ of a finite group G is an undirected simple graph whose vertex set is the set $\pi(G)$ of all prime divisors of $|G|$ and two vertices p and q are adjacent if and only if there exists an element of order pq in G. If $|G|$ is even then we denote by $\pi_1(G)$ the connected component of $\Gamma(G)$ containing 2. It is very known (see, for example, [1,2]) that the prime graph of any finite simple non-abelian group is not complete. We prove the following theorem which strengthen this result.

Theorem. *Let G be a finite simple non-abelian group. Then there exist in the graph $\Gamma(G)$ two nonadjacent odd vertices which do not divide $|Out(G)|$, moreover it is possible to take such vertices in $\pi_1(G)$, except when G is isomorphic to one of the following groups: M_{11}, M_{22}, J_1, J_2, J_3, HiS, A_n ($n \in \{5,6,7,9,12,13\}$), $A_1(q)$ ($q > 3$), $A_2^{\varepsilon}(q)$ ($q = p^m > 2$, p is a prime, $m \in \mathbb{N}$, $\varepsilon \in \{+,-\}$ and either $\pi(q+\varepsilon 1) = \{2\}$ or p divides $2m$), ${}^2A_3(3)$, ${}^2A_5(2)$, $C_3(2)$, $C_2(q)$ ($q > 2$), $D_4(2)$, ${}^2B_2(q)$ ($q = 2^{2k+1} > 2$), $G_2(q)$ ($q = 3^k$), $U_5(2)$.*

This work was supported by the Russian Foundation for Basic Research (project No. 13-01-00469), the Complex Program of UB RAS (project 15-16-1-5) and under the Agreement 02.A03.21.0006 of 27.08.2013 between the Ministry of Education and Science of the Russian Federation and Ural Federal University.

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The chromatic number of random Cayley graphs on the symmetric group

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In 2013 Noga Alon published the first pioneer work on the chromatic number of random Cayley graphs [1]. He considered the typical behavior of the chromatic number of a random Cayley graph of a given group of order n with respect to a randomly chosen set. This behavior depends on the group. General, cyclic and abelian groups were considered by Noga Alon. As open problems, he suggested consider more accurately the case of the symmetric group Sym_n.

In this talk we investigate bichromatic Cayley graphs $\Gamma = Cay(Sym_n, S)$ on the symmetric group Sym_n with a generating set S. The necessary and sufficient conditions of a Cayley graph Γ with the chromatic number $\chi(\Gamma) = 2$ are found.

Theorem 1. *Let $\Gamma = Cay(Sym_n, S)$ is a Cayley graph on the symmetric group Sym_n. Then Γ is bichromatic if and only if the generating set S does not contain even permutations.*

The proof is based on the classical Lagrange's theorem in group theory and the Kelarev's theorem [3], which describes all finite inverse semigroups with bipartite Cayley graphs.

Theorem 2. *Let a generating set S of a random Cayley graph $\Gamma = Cay(Sym_n, S)$ consists of k randomly chosen generators of Sym_n. If $n \geqslant 2$ and $k < \frac{n!}{2}$, then $\Gamma = Cay(Sym_n, S)$ is not, asymptotically almost surely, bichromatic.*

However, these results don't give the conditions for a random Cayley graph Γ to be connected.

Open problem *What are the necessary and sufficient conditions for $\Gamma = Cay(Sym_n, S)$ to be connected, where S is a randomly chosen generating set?*

In a particular case, when the generating set S of Γ is defined by reversals, the necessary and sufficient conditions of connectedness for Γ were found by Ting Chen and Steven Skiena in [2]. Let S consists of all reversals of fixed length ℓ. Then $\Gamma = Cay(Sym_n, S)$ is connected if and only if $\ell \equiv 2 (\text{mod } 4)$. In this case $|S| = n - \ell$ and the number of such generating sets is equal to $\lfloor \frac{n+1}{4} \rfloor$.

There are also two famous connected bichromatic Cayley graphs on the symmetric group known as the Star and the Bubble-sort graphs. These graphs are used for modelling interconnections networks [4].

The work has been supported by RFBS Grant 15-01-05867 and Grant NSh-1939.2014.1 of President of Russia for Leading Scientific Schools.

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On Cameron's question about primitive permutation groups with stabilizer of two points that is normal in the stabilizer of one of them

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Cameron formulated the following question (see [1], [3, question 9.69]). Assume that G is a primitive permutation groups on a finite set X, $x \in X$ and G_x acts regularly on the G_x-orbits $G_x(y)$ containing y (i.e. G_x induces on $G_x(y)$ a regular permutation group). Is it true that this action is faithful, i.e., that $|G_x| = |G_x(y)|$? Note that the question on the faithfulness of the action of a stabilizer G_x on a regular suborbit $G_x(y)$ was also treated earlier (see [5], [6], [7]).

It is clear that the regularity of the action of the group G_x on $G_x(y)$ is equivalent to the property $G_{x,y} \trianglelefteq G_x$, and the equality $|G_x| = |G_x(y)|$ is equivalent to the equality $G_{x,y} = 1$. Thus, Cameron's question is equivalent to the question on the fulfilment for an arbitrary primitive permutation group G on a finite set X of the following property.

(Pr) If $x \in X$ and $y \in X \setminus \{x\}$, then $G_{x,y} \trianglelefteq G_x$ implies $G_{x,y} = 1$.

Obviously, Cameron's question is also equivalent to the question on the fulfilment for an arbitrary finite group G of the following property.

(Pr*) If M_1 and M_2 are different conjugate maximal subgroups in G, then $M_1 \cap M_2 \trianglelefteq M_1$ implies $M_1 \cap M_2 \trianglelefteq G$.

In the present work (using [2]), we prove the following theorem.

Theorem. *Let G be a primitive permutation group on a finite set X and $x \in X$. Assume that the socle of G is not isomorphic to power of an exceptional group T of Lie type $E_8(q)$ with T_x of type* (d) *or* (e) *from* [4]*. Then the permutation group G satisfies property* **(Pr)**. *In particular, for such primitive groups G, the answer to Cameron's question is positive.*

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The automorphism group of finite semifield

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A *semifield* is an algebraic structure $\langle W, +, \circ \rangle$, satisfying the following axioms:
1) $\langle W, + \rangle$ is abelian group;
2) $\langle W^*, \circ \rangle$ is a loop;
3) $x \circ (y + z) = x \circ y + x \circ z$ and $(y + z) \circ x = y \circ x + z \circ x$ for all $x, y, z \in W$.

The projective plane π coordinatizing by semifield W is called a *semifield plane.* Let π be a semifield plane of order p^n, p be prime. We can represent the coordinatizing semifield of such a plane as a n-dimensional linear space over $\mathbb{Z}_p$, with multiplication law

$$x \circ y = x\theta(y), \quad x, y \in W.$$

Here $\theta : W \to GL_n(p) \cup \{0\}$ is a bijective mapping, satisfying the conditions:
1) $\theta(y + z) = \theta(y) + \theta(z)\ \forall y, z \in W$;
2) $\theta(0, 0, \ldots, 0, 0) = 0$, $\theta(0, 0, \ldots, 0, 1) = E$ (identity matrix).

We shall call the matrix set $R = \{\theta(y) | y \in W\}$ a *regular set.*

Theorem. *The bijective mapping $x \to xA$, $x \in W$, is an automorphism of semifield W for $A \in GL_n(p)$ if and only if*

$$A^{-1}\theta(y)A = \theta(yA) \qquad \forall y \in W.$$

Moreover, the matrix

$$\begin{pmatrix} A & 0 \\ 0 & A \end{pmatrix}$$

determines the collineation of semifield plane π, that fixes a triangle $(0, 0)$, (0), (∞) and a line $y = x$.

We used the matrix representation of automorphism to construct the autotopism subgroup of semifield plane and automorphism group of coordinatizing semifield of some small orders, odd and even. Also the matrix representation of inner automorphisms [1] of finite semifield is determined.

The author was supported by Russian Fund of Fundamental Researches, grant 15-01-04897 A.

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An injective map from the set of maximum independent sets in a Doob graph to the set of 4-ary distance-2 MDS codes

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The Cartesian product $D(m,n) \stackrel{\text{def}}{=} \mathrm{Sh}^m \times K_4^n$ of m copies of the Shrikhande graph Sh (see the left part of Fig. 1) and n copies of the complete graph K_q of order $q=4$ is called a Doob graph if $m>0$, while $D(0,n)$ is the Hamming graph $H(n,4)$ (in general $H(n,q) \stackrel{\text{def}}{=} K_q^n$). The Doob graph $D(m,n)$ is a distance-regular graph with the same parameters as $H(2m+n,4)$. It is easy to see that the independence number of this graph is 4^{2m+n-1}. The maximum independent sets in the Hamming graphs are known as the distance-2 MDS codes, or the Latin hypercubes (in the last case, one coordinate is usually considered as a function of the other coordinates). It is natural to generalize these notions to the maximum independent sets in Doob graphs; however, for generalized Latin hypercubes in $D(m,n)$, we need at least one K_4 coordinate, i.e., $n>0$. There are 4 trivial MDS codes in $D(0,1)$; 24 equivalent distance-2 MDS codes in $D(0,2)$ (16 of them can be found in Fig. 1); 16 distance-2 MDS codes in $D(1,0)$ (see Fig. 1), which form two equivalence classes.

The goal of the current correspondence is to describe a rather simple recursive way to map injectively the set $\mathrm{MDS}_{m,n}$ of distance-2 MDS codes in $D(m,n)$ into $\mathrm{MDS}_{0,2m+n}$. At first, we define the map κ from $\mathrm{MDS}_{1,0}$ into $\mathrm{MDS}_{0,2}$, see Fig. 1. This map has the following important property: two MDS codes M' and M'' in $D(1,0)$ intersect if and only if their images $\kappa M'$ and $\kappa M''$ intersect. It follows that κ:

$$\kappa M \stackrel{\text{def}}{=} \big\{(x_1,...,x_m,z_1,z_2,y_1,...,y_n) \in D(m,n+2) \,\big|\, (z_1,z_2) \in \kappa\{v \in \mathrm{Sh} \mid (x_1,...,x_m,v,y_1,...,y_n) \in M\}\big\}$$

maps $\mathrm{MDS}_{m+1,n}$ into $\mathrm{MDS}_{m,n+2}$. Then, κ^m maps $\mathrm{MDS}_{m,n}$ into $\mathrm{MDS}_{0,2m+n}$. A constructive characterization of the class $\mathrm{MDS}_{0,2m+n}$ can be found in [1]; using the map κ^m, it is possible to extract some information on $\mathrm{MDS}_{m,n}$ for arbitrary m. In particular, $|\mathrm{MDS}_{m,n}| = 2^{2^{2m+n}(1+o(1))}$ (by comparison, the number of all vertex subsets in $D(m,n)$ is $2^{2^{4m+2n}}$).

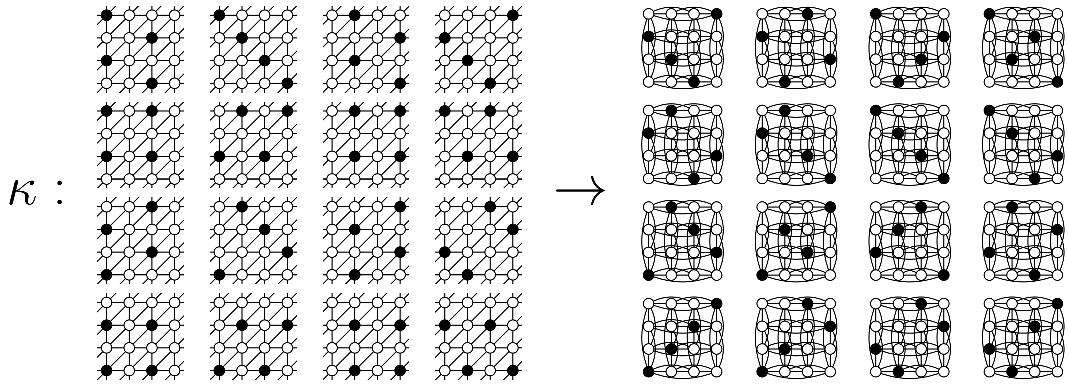

Figure 1: The 16 maximum independent sets in Sh and the corresponding independent sets in K_4^2

This research was funded by the Russian Science Foundation (grant No 14-11-00555).

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Compositional structure of groups isospectral go $U_3(3)$

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In this work only finite groups are studied. The *spectrum* $\omega(G)$ of a group G is the set of its element orders. By a *section* of G we mean a quotient group H/N, where $N, H \leq G$ and $N \trianglelefteq H$. Groups G and H are called *isospectral*, if $\omega(G) = \omega(H)$. Let ω be a subset of natural numbers. Following [1], we call a group G *critical with respect to* ω (or ω*-critical*), if ω coincides with the spectrum of G and does not coincide with the spectrum of any proper section of G.

If a simple group L has infinitely many groups isospectral to L, then it is important to study critical groups isospectral to L. In [2, 3] the complete description is given of critical groups isospectral to non-abelian simple alternating and sporadic groups and also the special linear group $SL_3(3)$.

In this work we study groups critical with respect to the spectrum of the projective special unitary group $U_3(3)$. In particular, we prove the following

Theorem. *Let G be a group isospectral to $U_3(3)$ that contains a normal subgroup N, such that $G/N \simeq PGL_2(7)$. Then N is a 2-group and every G-chief factor of N is isomorphic to a 6-dimensional module of the group $PGL_2(7)$. Also $G = NH$ for some subgroup $H \simeq PGL_2(7)$. If in addition G is critical with respect to $\omega(U_3(3))$, then $|N| = 2^6$.*

Moreover, H has a representation $\langle a, b, c \mid a^2 = b^3 = c^2 = (ab)^7 = (ac)^2 = (bc)^2 = [a, b]^4 = 1\rangle$ and if we regard N as a vector space over $GF(2)$ then a base of N can be chosen in such a way that the action of H on N is defined by the following matrices:

$$a \sim \begin{pmatrix} 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot & \cdot & \cdot \\ 1 & \cdot & 1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot & 1 \end{pmatrix}, \; b \sim \begin{pmatrix} \cdot & 1 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot & \cdot & \cdot \\ 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & 1 \\ \cdot & \cdot & \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 & \cdot \end{pmatrix}, \; c \sim \begin{pmatrix} \cdot & \cdot & \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & 1 \\ 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot & \cdot & \cdot \\ \cdot & & \cdot & 1 & \cdot & \cdot \;\cdot \end{pmatrix}.$$

This work was partially supported by RFBR Grants 13-01-00505 and 14-01-90013.

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On some groups of period 12

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In this talk we consider groups of period 12. In particular, we find conditions that guarantee local finiteness of such groups.

It is well-known that groups of period 4 and period 6 are locally finite [1–4]. In [1, 5–7] local finiteness of groups of period 12 was proved under some additional conditions.

Our goal is to reduce a question whether a group of period 12 is locally finite to a question whether its subgroups generated by three elements of order 3 are finite. Our main result is stated in the following theorem.

Theorem. *A group of period* 12 *is locally finite if and only if every subgroup* H *of* G *is finite, given that* H *satisfies one of the following conditions.*

1. H *is generated by an element* a *of order* 3 *and elements* b *and* c *of order* 2*, such that* $(ab)^3 = (bc)^3 = 1$.

2. H *is generated by elements* a *and* b *of order* 3 *and an element* c *of order* 2*, such that* $(ac)^2 = 1$.

In particular, a group of period 12 *is locally finite if every of its subgroups generated by three elements of order* 3 *is finite.*

The work was supported by Russian Foundation for Basic Research, Grants 13-01-00505 and 14-01-90013.

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Strongly regular graphs with nonprincipal eigenvalue 5 and its extensions

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We consider nondirected graphs without loops and multiple edges. For vertex a of a graph Γ the subgraph $\Omega_i(a) = \{b \mid d(a,b) = i\}$ is called i-neighborhood of a in Γ. We set $[a] = \Gamma_1(a)$, $a^\perp = \{a\} \cup [a]$.

Degree of an vertex a of Γ is the number of vertices in $[a]$. Graph Γ is called regular of degree k, if the degree of any vertex is equal k. The graph Γ is called amply regular with parameters (v,k,λ,μ) if Γ is regular of degree k on v vertices, and $|[u] \cap [w]|$ is equal λ, if u adjacent to w, is equal μ, if $d(u,w) = 2$. Amply regular graph of diameter 2 is called strongly regular.

A partial geometry $pG_\alpha(s,t)$ is a geometry of points and lines such that every line has exactly $s+1$ points, every point is on $t+1$ lines (with $s > 0,\ t > 0$) and for any antiflag (P,y) there are exactly α lines z_i containing P and intersecting y. In the case $\alpha = 1$ we have generalized quadrangle $GQ(s,t)$.

Jack Koolen suggested the problem investigation of distance-regular graphs whose local subgraphs are strongly regular graphs with the second eigenvalue at most t for some natural number t. In [1] the solving of Koolen problem in the case $t = 3$ was began.

We begin the investigation of the case $t = 5$.

Strongly regular graph Γ with the second eigenvalue $m-1$ is called exceptional if Γ does not belong the following list:

(1) the union of isolated m-cliques;

(2) pseudogepmetric graph for $pG_t(t+m-1,t)$;

(3) the complement of pseudogepmetric graph for $pG_m(s,m-1)$;

(4) conference graph with parameters $(4\mu+1, 2\mu, \mu-1, \mu)$, $\sqrt{4\mu+1} = m-1$.

In this paper it is obtained reduction to locally exceptional graphs.

Theorem. *Let Γ be a distance-regular graph with strongly regular local subgraphs having the second eigenvalue t, $4 < t \leq 5$, u is a vertex of Γ. Then $[u]$ is an exceptional strongly regular graph, or one of the following holds:*

(1) $[u]$ *is the union of isolated* 6*-cliques;*

(2) $[u]$ *is the pseudogepmetric graph for* $pG_{s-5}(s,s-5)$ *and either*

(*i*) Γ *is strongly regular graph with parameters* $(176,49,12,14)$, $(209,100,45,50)$, $(806,625,480,500)$, $(1464,1225,1020,1050)$*, and* $s = 6,9,24,34$ *respectively, or*

(*ii*) $s = 6$ *and* Γ *is Johnson graph* $J(14,7)$*, or its standard quotient or graph with intersection array* $\{49,36,1;1,12,49\}$*, or*

(*iii*) $s = 7$ *and* Γ *has intersection array* $\{64,42,1;1,21,64\}$*, or*

(*iv*) $s = 10$ *and* Γ *is Taylor graph;*

(3) $[u]$ *the complement of pseudogepmetric graph for* $pG_6(s,5)$*,* Γ *is strongly regular graph with parameters* $(259,42,5,7)$*,* $(356,85,30,17)$*, and* $s = 8,6$ *respectively, or* $s = 12$ *and* Γ *is Taylor graph;*

(4) $[u]$ *is the conference graph with parameters* $(4l+1, 2l, l-1, l)$*,* $l \in \{21,22,24,25,27,28,29,30\}$ *and* Γ *is Taylor graph.*

This work was supported by the grant of Russian Science Foundation, project no. 15-11-10025.

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Automorphisms of distance-regular graph with intersection array $\{204, 175, 48, 1; 1, 12, 175, 204\}$

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We consider nondirected graphs without loops and multiple edges. For vertex a of a graph Γ the subgraph $\Omega_i(a) = \{b \mid d(a,b) = i\}$ is called i-neighborhood of a in Γ. We set $[a] = \Gamma_1(a)$.

Degree of an vertex a of Γ is the number of vertices in $[a]$. Graph Γ is called regular of degree k, if the degree of any vertex is equal k. The graph Γ is called amply regular with parameters (v, k, λ, μ) if Γ is regular of degree k on v vertices, and $|[u] \cap [w]|$ is equal λ, if u adjacent to w, is equal μ, if $d(u,w) = 2$. Amply regular graph of diameter 2 is called strongly regular.

Distance-regular graph Γ with intersection array $\{204, 175, 48, 1; 1, 12, 175, 204\}$ is $AT4(4,6,5)$-graph [1]. Antipodal quotient $\bar{\Gamma}$ has parameters $(800, 204, 28, 60)$.

In this paper automorphisms of distance-regular graph Γ with intersection array $\{204, 175, 48, 1; 1, 12, 175, 204\}$ and of antipodal quotient $\bar{\Gamma}$ are investigated.

Theorem 1. *Let Γ be a strongly regular with parameters $(800, 204, 28, 60)$, $G = \mathrm{Aut}(\Gamma)$, g be an element of prime order p of G and $\Omega = \mathrm{Fix}(g)$. Then $\pi(G) \subseteq \{2, 3, 5, 7, 17\}$ and one of the following holds:*

(1) *Ω is empty graph, either $p = 5$, $\alpha_1(g) = 200l$, or $p = 2$ and $\alpha_1(g) = 40m$;*

(2) *Ω is n-clique, either $p = 17$, $n = 1$ and $\alpha_1(g) = 204$, or $p = 5$, $n = 5$ and $\alpha_1(g) = 200s + 20$ or $p = 7$, $n = 2$ and $\alpha_1(g) = 280t + 168$;*

(3) *Ω is l-coclique, either $p = 3$, $l = 3m + 2$ and $\alpha_1(g) = 120t + 12m + 48$, or $p = 2$, $\alpha_1(g) = 80t + 4l$, where $l = 8, 10, ..., 92$;*

(4) *Ω is the union of m isolated 5-cliques, $2 \le m \le 5$, $\alpha_1(g) = 200s + 20m$;*

(5) *Ω contains geodesic 2-way and either*

(*i*) *$p = 3$, Ω is the union of $3m + 1$ isolated subgraphs $K_{4\times 2}$ and $\alpha_1(g) = 96m + 120t + 72$, or*

(*ii*) *$p = 2$, $|\Omega| = 2l \le 240$, $\lambda_\Omega = 0, 2, ..., 26$, degrees of vertices in Ω equal $0, 2, ..., 34$ and $\alpha_1(g) = 80t + 8l$.*

Theorem 2. *Let Γ be a distance-regular graph Γ with intersection array $\{204, 175, 48, 1; 1, 12, 175, 204\}$, $G = \mathrm{Aut}(\Gamma)$, g be an element of prime order p of G and $\Omega = \mathrm{Fix}(g)$ contains s vertices in t antipodal classes. Then $\pi(G) \subseteq \{2, 5, 7, 17\}$ and one of the following holds:*

(1) *Ω is empty graph, either $p = 5$, $\alpha_1(g) = 200(4+m-l)$, $\alpha_2(g) = 1000l$ and $\alpha_3(g) = 200(16-m-4l)$, or $p = 2$, $\alpha_1(g) = 80(4 + m - l)$, $\alpha_2(g) = 400l$ and $\alpha_3(g) = 80(46 - m - 4l)$;*

(2) *g induces trivial automorphism of antipodal quotient $\bar{\Gamma}$, $p = 5$ and $\alpha_4(g) = v$;*

(3) *Ω is the antipodal class of Γ, $p = 17$, $\alpha_1(g) = 340 + 680n$, $\alpha_2(g) = 2975$ and $\alpha_3(g) = 680(1 - n)$;*

(4) *Ω is the union of two antipodal classes, $p = 7$, $\alpha_1(g) = 910 + 280n - 70l$, $\alpha_2(g) = 350l$, $\alpha_3(g) = 3080 - 280l - 280n$, $l = 1, 5, 9$;*

(5) *$p = 5$, $t = 5, s = 5$, $\alpha_1(g) = 700 + 200(m - l)$, $\alpha_2(g) = 1000l - 125$ and $\alpha_3(g) = 200(17 - m - 4l)$.*

Corollary. *Let Γ be a distance-regular graph Γ with intersection array $\{204, 175, 48, 1; 1, 12, 175, 204\}$. Then group $G = \mathrm{Aut}(\Gamma)$ is solvable.*

This work was supported by the grant of Russian Science Foundation, project no. 15-11-10025.

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Strongly regular graphs with strongly regular local subgraphs having second eigenvalue 5

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We consider nondirected graphs without loops and multiple edges. For vertex a of a graph Γ the subgraph $\Omega_i(a) = \{b \mid d(a,b) = i\}$ is called i-neighborhood of a in Γ. We set $[a] = \Gamma_1(a)$, $a^\perp = \{a\} \cup [a]$.

Degree of an vertex a of Γ is the number of vertices in $[a]$. Graph Γ is called regular of degree k, if the degree of any vertex is equal k. The graph Γ is called amply regular with parameters (v, k, λ, μ) if Γ is regular of degree k on v vertices, and $|[u] \cap [w]|$ is equal λ, if u adjacent to w, is equal μ, if $d(u,w) = 2$. Amply regular graph of diameter 2 is called strongly regular.

A partial geometry $pG_\alpha(s,t)$ is a geometry of points and lines such that every line has exactly $s+1$ points, every point is on $t+1$ lines (with $s > 0,\ t > 0$) and for any antiflag (P, y) there are exactly α lines z_i containing P and intersecting y. In the case $\alpha = 1$ we have generalized quadrangle $GQ(s,t)$.

Jack Koolen suggested the problem investigation of distance-regular graphs whose local subgraphs are strongly regular graphs with the second eigenvalue at most t for some natural number t. Recently this problem was solved for $t = 3$. At present near finishing the case $t = 4$. We begin the investigation of the case $t = 5$. In [1] was obtained the reduction to the exceptional local subgraphs. Let Γ be a distance regular graph of diameter $d \geq 3$. Then $c_2 \leq b_1$. A. Makhnev and D. Paduchikh found parameters of exceptional strongly regular graphs with the second eigenvalue 5, which may be local subgraphs in amply regular graphs with $\mu \leq b_1$.

In this paper it is determined parameters of strongly regular graphs with strongly regular local subgraphs having the second eigenvalue 5.

Theorem. *Let Γ be a strongly regular graph with strongly regular local subgraphs having the second eigenvalue* 5. *Then Γ has parameters* $(176, 49, 12, 14)$, $(209, 100, 45, 50)$, $(259, 42, 5, 7)$, $(356, 85, 30, 17)$, $(806, 625, 480, 500)$, $(1464, 1225, 1020, 1050)$ *or local subgraphs are exceptional and Γ has parameters*

(1) $(100, 36, 14, 12)$, $(100, 77, 60, 56)$, $(189, 100, 55, 50)$, $(169, 112, 75, 72)$, $(330, 105, 40, 30)$, $(345, 120, 35, 45)$, $(400, 210, 110, 110)$, $(512, 133, 24, 38)$, $(550, 225, 80, 100)$, $(560, 325, 180, 200)$, $(605, 280, 117, 140)$, $(680, 175, 30, 50)$, $(846, 260, 70, 84)$, $(946, 273, 80, 78)$, $(990, 345, 120, 120)$,

(2) $(1003, 300, 65, 100)$, $(1016, 259, 42, 74)$, $(1036, 375, 110, 150)$, $(1080, 260, 70, 60)$, $(1090, 441, 152, 196)$, $(1122, 209, 16, 44)$, $(1199, 550, 225, 275)$, $(1200, 605, 280, 330)$, $(1458, 329, 40, 84)$, $(1520, 385, 60, 110)$, $(1577, 400, 105, 100)$, $(1976, 175, 30, 14)$;

(3) $(2025, 680, 175, 255)$, $(2032, 1275, 770, 850)$, $(2034, 437, 100, 92)$, $(2209, 624, 161, 182)$, $(2420, 885, 260, 360)$, $(2508, 1199, 550, 594)$, $(2809, 540, 77, 110)$, $(3250, 1305, 440, 580)$, $(3481, 960, 245, 272)$, $(3844, 630, 68, 110)$, $(3872, 343, 54, 28)$, $(3888, 1625, 580, 750)$, $(3950, 385, 60, 35)$;

(4) $(4256, 259, 42, 14)$, $(4418, 637, 96, 91)$, $(4496, 1015, 150, 252)$, $(4512, 650, 55, 100)$, $(4706, 3625, 2760, 2900)$, $(4941, 1520, 385, 504)$, $(5074, 969, 176, 187)$, $(5625, 1520, 385, 420)$, $(5820, 2783, 1270, 1386)$, $(7139, 3250, 1305, 1625)$, $(7280, 1015, 150, 140)$, $(9801, 1600, 205, 272)$.

This work was supported by the grant of Russian Science Foundation, project no. 15-11-10025.

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On the realizability of some graphs as Gruenberg–Kegel graphs of finite groups

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We use the term "group" while meaning "finite group" and the term "graph" while meaning "undirected graph without loops and multiple edges".

Let G be a group. Denote by $\pi(G)$ the set of all prime divisors of the order of G and by $\omega(G)$ the *spectrum* of G, i.e., the set of all its element orders. The set $\omega(G)$ defines the *Gruenberg–Kegel graph* (or the *prime graph*) $\Gamma(G)$ of G; in this graph, the vertex set is $\pi(G)$ and different vertices p and q are adjacent if and only if $pq \in \omega(G)$.

We say that a graph Γ with $|\pi(G)|$ vertices *is realizable as the Gruenberg–Kegel graph of a group G* if there exists a marking the vertices of Γ by different primes from $\pi(G)$ such that the marked graph is equal to $\Gamma(G)$. A graph Γ *is realizable as the Gruenberg–Kegel graph of a group* if Γ is realizable as the Gruenberg–Kegel graph of an appropriate group G.

The following problem arises.

Problem. *Let Γ be a graph. Is Γ realizable as the Gruenberg–Kegel graph of a group*?

Of course, in general, the problem has negative solution. For example, the graph consisting of five pairwise non-adjacent vertices (5-coclique) is not realizable as the Gruenberg–Kegel graph of a group.

In this talk, we will tell on the realizability of some graphs as Gruenberg–Kegel graphs of groups. In particular, we prove the following theorem.

Theorem. *Let Γ be a complete bipartite graph $K_{m,n}$, where $m \leq n$. Then Γ is realizable as the Gruenberg–Kegel graph of a group if and only if $m + n \leq 6$ and $(m, n) \neq (3, 3)$.*

The distribution of cycles of length $O(n)$ in the Star graph

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The Star graph $S_n = Cay(Sym_n, ST), n \geqslant 2$ is a Cayley graph on the symmetric group Sym_n with the generating set of transpositions $ST = \{t_i \in Sym_n, 2 \leqslant i \leqslant n\}$ exchanging i'th element of the permutation with the first. Graph S_n, $n \geqslant 3$, is bipartite, therefore contains only even cycles of lengths C_l, where $6 \leqslant l \leqslant n!$ [1] and has the diameter $D = \lfloor \frac{3(n-1)}{2} \rfloor$.

The current work continues the study of cyclic structure of the Star graph, started in [2], under a different approach. The distribution and the structure of vertices at each distance layer d, where $1 \leqslant d \leqslant D$, from the identity vertex is known [3]. We employ this result to study the number of cycles of lengths $2d$, $3 \leqslant d \leqslant D$, constructed from two non-intersecting shortest paths to the vertex at distance d from the identity vertex. The study of such cycles is closely related to the method proposed to solve the First Passage Percolation problem on graphs [4, 5].

Any permutation $\pi \in Sym_n$ can be represented uniquely in terms of non-intersecting cycles, i.e.

$$\pi = (1\, \pi_2^1 \dots \pi_{l_1}^1)(\pi_1^2 \dots \pi_{l_2}^2) \dots (\pi_1^k \dots \pi_{l_k}^k).$$

Denote the cycle of length l containing the element "1" as $l - CO$ and not containing it as $l - CN$, then the vertices on the distance layer d may have either

1. only a $(d+1) - CO$;
2. an $m - CO$, $1 \leqslant m \leqslant d-2$ and $k \geqslant 1$ items of $l_i - CN$, where $1 \leqslant i \leqslant k$, such that $d = k + (m-1) + \sum_{i=1}^{k} l_i$.

The following theorems describe the distribution of distinct cycles in the Star graph S_n for $3 \leqslant d \leqslant D$.

Theorem 1. *The number of cycles of length $2d$ passing through the vertices with $1 - CO$ and $k \geqslant 2$ items of $l_i - CN$, over all $k + \sum_{i=1}^{k} l_i = d$, is*

$$N_{C_1} = O\left(k!(d-3k-2)^{4k-2} + k!(d-3k-2)^{3k-1}\right)(n-1)\dots(n-d+k).$$

Theorem 2. *The number of cycles of length $2d$ passing through the vertices with $m - CO$ and $k \geqslant 2$ items of $l_i - CN$, over all $m - 1 + k + \sum_{i=1}^{k} l_i = d$, is*

$$N_{C_2} = O\left((k!)^2(d-3k-3)^{4k-2}\right)(n-1)\dots(n-d+k).$$

The work has been supported by RFBS Grant 15-01-05867 and Grant NSh-1939.2014.1 of President of Russia for Leading Scientific Schools.

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Invariants of virtual links

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Virtual knot theory has been introduced by Kauffman [1] as a generalization of the classical knot theory. Virtual knots (and links) are represented as generic immersions of circles in the plane (virtual link diagrams) where double points can be classical (with the usual information on overpasses and underpasses) or virtual. Virtual link diagrams are equivalent under ambient isotopy and some types of local moves (generalized Reidemeister moves).

Using virtual generalized Reidemeister moves we can introduce a notion of "virtual"braids. Virtual braids on n strands form a group denoted by VB_n. The relation between virtual braids and virtual knots (and links) are completely determined by a generalization of Alexander and Markov Theorem [2, 3]. It is worth to mention that for virtual braids an Alexander-like theorem states that any virtual link can be represented as the closure of a virtual braid.

In the classical case it is known that the braid group embeds into $\mathrm{Aut}(F_n)$ by Artin representation which is a local one. Wada [6] classified all local representations of the braid group B_n into $\mathrm{Aut}(F_n)$. There are four types. It is proved [7] that these representations are faithful.

Proposition *For every Wada representation*

$$w_1^r, w_2, w_3, w_4 : B_n \to AutF_n,\ r \in \mathbb{Z}$$

it is possible to construct the corresponding representation of the virtual braid group

$$W_1^r, W_2, W_3, W_4 : VB_n \to AutF_{n+1},\ r \in \mathbb{Z},$$

such that the restriction each of them onto B_n is coincide with the corresponding Wada representation, i.e.

$$W_k|_{B_n} = w_k,\ k = 1, 2, 3, 4.$$

Analogously to the way shown in [4] we introduce the notion of the group of the virtual link $G(vL)$ for representations of the virtual braid group $W_k,\ k = 1, \ldots, 4$. Let $vL = \widehat{\beta_v}$ be a closure of the virtual braid, where $\widehat{\beta_v} \in VB_n$. We define

$$G_k(vL) = \langle x_1, x_2, \ldots, x_n, y \parallel x_i = W_k(\beta_v)(x_i),\ i = 1, 2, \ldots, n\rangle,\ k = 1, \ldots, 4.$$

Theorem. *The constructed groups $G_k(vL),\ k = 1, \ldots 4,$ are invariants of the virtual link vL.*

Acknowledgements. The research was supported by the Russian-Chinese grant (14-01-91161).

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Three-dimensional homogeneous spaces with invariant affine connections

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Let $(\overline{G}, M)$ be a three-dimensional homogeneous space. We fix an arbitrary point $o \in M$ and denote by $G = \overline{G}_o$ the stationary subgroup of o. Since we are interested only the local equivalence problem, we can assume without loss of generality that both $\overline{G}$ and G are connected. Then we can correspond the pair $(\bar{\mathfrak{g}}, \mathfrak{g})$ of Lie algebras to $(\overline{G}, M)$, where $\bar{\mathfrak{g}}$ is the Lie algebra of $\overline{G}$ and $\mathfrak{g}$ is the subalgebra of $\bar{\mathfrak{g}}$ corresponding to the subgroup G. This pair uniquely determines the local structure of $(\overline{G}, M)$, that is two homogeneous spaces are locally isomorphic if and only if the corresponding pairs of Lie algebras are equivalent. A pair $(\bar{\mathfrak{g}}, \mathfrak{g})$ is *effective* if $\mathfrak{g}$ contains no non-zero ideals of $\bar{\mathfrak{g}}$, a homogeneous space $(\overline{G}, M)$ is locally effective if and only if the corresponding pair of Lie algebras is effective.

An *isotropic* $\mathfrak{g}$*-module* $\mathfrak{m}$ is the $\mathfrak{g}$-module $\bar{\mathfrak{g}}/\mathfrak{g}$ such that

$$x.(y+\mathfrak{g}) = [x, y]+\mathfrak{g}.$$

The corresponding representation $\lambda\colon \mathfrak{g} \to \mathfrak{gl}(\mathfrak{m})$ is called an *isotropic representation* of $(\bar{\mathfrak{g}}, \mathfrak{g})$. The pair $(\bar{\mathfrak{g}}, \mathfrak{g})$ is said to be *isotropy-faithful* if its isotropic representation is injective. Invariant affine connections on $(\overline{G}, M)$ are in one-to-one correspondence [1] with linear mappings $\Lambda\colon \bar{\mathfrak{g}} \to \mathfrak{gl}(\mathfrak{m})$ such that $\Lambda|_{\mathfrak{g}} = \lambda$ and Λ is $\mathfrak{g}$-invariant. We call this mappings *(invariant) affine connections* on the pair $(\bar{\mathfrak{g}}, \mathfrak{g})$. If there exists at least one invariant connection on $(\bar{\mathfrak{g}}, \mathfrak{g})$ then this pair is isotropy-faithful [2]. We find all of this pairs. The curvature and torsion tensors of the invariant affine connection Λ are given by the following formulas:

$$R\colon \mathfrak{m} \wedge \mathfrak{m} \to \mathfrak{gl}(\mathfrak{m}),\ (x_1+\mathfrak{g}) \wedge (x_2+\mathfrak{g}) \mapsto [\Lambda(x_1), \Lambda(x_2)] - \Lambda([x_1, x_2]);$$

$$T\colon \mathfrak{m} \wedge \mathfrak{m} \to \mathfrak{m},\ (x_1+\mathfrak{g}) \wedge (x_2+\mathfrak{g}) \mapsto \Lambda(x_1)(x_2+\mathfrak{g}) - \Lambda(x_2)(x_1+\mathfrak{g}) - [x_1, x_2]_{\mathfrak{m}}.$$

We restate the theorem of Wang on the holonomy algebra of an invariant connection: the Lie algebra of the holonomy group of the invariant connection defined by $\Lambda : \bar{\mathfrak{g}} \to \mathfrak{gl}(3, \mathbb{R})$ on $(\bar{\mathfrak{g}}, \mathfrak{g})$ is given by

$$V + [\Lambda(\bar{\mathfrak{g}}), V] + [\Lambda(\bar{\mathfrak{g}}), [\Lambda(\bar{\mathfrak{g}}), V]] + \dots,$$

where V is the subspace spanned by $\{[\Lambda(x), \Lambda(y)] - \Lambda([x, y]) | x, y \in \bar{\mathfrak{g}}\}$.

We describe all local three-dimensional homogeneous spaces, allowing affine connections, it is equivalent to the description of effective pairs of Lie algebras, and all invariant affine connections on the spaces together with their curvature, torsion tensors and holonomy algebras. We use the algebraic approach for description of connections, methods of the theory of Lie groups, Lie algebras and homogeneous spaces.

The results of work can be used in research work on the differential geometry, differential equations, topology, in the theory of representations, in the theoretical physics. In particular, the results can find practical application in general theory of relativity, which, with mathematical point of view, is based on the geometry of the curved spaces, in the nuclear physics and physics of elementary particles that are associated with geometric interpretation of equations. Methods stated in the work, can be applied for the analysis of physical models, and algorithms classification of homogeneous spaces, affine connections on these spaces, curvature and torsion tensors, holonomy algebras can be computerized and used for the decision of similar problems in large dimensions.

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On a generalization of relations schemas, related to groups $U_3(q)$ and ${}^2G_2(3^{2l+1})$

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Let X be a finite set, R_i $(i = 0, 1, \ldots, d)$ – binary relations on X, that satisfy conditions of *symmetric commutative associative scheme of relations on d classes* (see definitions in [1]), except condition of constancy number of intersections p^1_{ij}. Pair $(X, \{R_i\}_{\{0\leqslant i\leqslant d\}}) = \mathcal{X}(X)$ is called *scheme*, $\Gamma^{(i)} = (X, R_i)$ – *graph of i-th relation*. Scheme $\mathcal{X}(X)$ is called *scheme of cliques* if:

1. Graph $\Gamma^{(1)}$ of 1st relation is disconnected with $n + 1$ connected components – cliques K_1, K_2, …, K_{n+1} with $d - 1$ vertices each.

2. If x is fixed vertex of clique K_s, $K_t \neq K_s$, y iterates over vertices from K_t, then in $(x, y) \in R_i$ index i iterates over indices from $2, 3, \ldots, d$.

If scheme of cliques satisfies the additional condition

3. If i, j, k and s, t, l are arbitrary sets of pairwise distinct indices from $\{2, 3, \ldots, d\}$, then $p^k_{ij} = p^l_{st}$, $p^k_{ii} = p^l_{ss}$, $p^i_{ii} = p^s_{ss}$, and $p^1_{ij} \in \{0, r\}$, $r \neq 0$,

then it called scheme of cliques with the *absolute number of intersections*.

In [2] was announced existence of these schemes on the class X of conjugate elements of prime order p from centers of p-Sylow subgroups in group $G \in \{L_2(q), Sz(q), U_3(q)\}$ (with even $q \geqslant 4$ in last case) with $q = p^m$, where p is prime. Also was announced distance-regularity with array of intersections $\{n, n - p^i_{ii} - 1, 1; 1, p^k_{ii}, n\}$ of their graphs $\Gamma^{(i)}$ i-th relations with $i \in \{2, 3, \ldots, d\}$, $k \neq i$, where $p^k_{ii} = p^i_{ii} = (n - 1)/(d - 1)$. The following theorem was proved.

Theorem. *If $G \in \{U_3(q), {}^2G_2(q)\}$, $q = p^m$ – degree of odd prime number p with $q = 3^{2l+1}$ in case $G = {}^2G_2(q)$, X – class of conjugate elements of order p from centers of p-Sylow subgroup of group G, then on X can be defined relations R_i $(i = 0, 1, \ldots, d)$, such that $(X, \{R_i\}_{\{0\leqslant i\leqslant d\}})$ – scheme of cliques with $d = q$, $n = q^3$ и $p^k_{ij} = p^k_{ii} = p^i_{ii} = 2(q^2 + q + 1)$ for all $i, j, k \in \{2, 3, \ldots, d\}$.*

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Formations of finite groups and Hawkes graph

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All groups considered are finite. There have been a lot of papers recently in which with every finite group associates certain graph. The considered problem was to analyze the relations between the structure of a group and the properties of its graph. This trend goes back to 1878 when A. Cayley [1] introduced his graph.

Let $\pi(G)$ be the set of prime divisors of $|G|$. Recall [2] that the Gruenberg-Kegel or the prime graph Γ_p of a group G is the graph with the vertex set $\pi(G)$ and (p,q) is an edge if and only if G contains element of order pq. This graph is connected to the problem of recognition of groups by their graph. Recall that a group G is called recognizable by the prime graph if $\Gamma_p(G) = \Gamma_p(H)$ implies $H \simeq G$ for any group H. There are many non-isomorphic groups with nontrivial solvable radical and the same prime graph. That is why of prime interest (for example see [3]) is this problem only for simple and almost simple groups. In this paper we will consider the recognition problem up to a class of groups.

Definition 1. *A function $\Gamma : \{groups\} \to \{graphs\}$ is called graph function.*

Definition 2. *Let Γ be a graph function and $\mathfrak{X}$ be a class of groups. We shall say that $\mathfrak{X}$ is recognized by Γ if from $G_1 \in \mathfrak{X}$ and $\Gamma(G_1) = \Gamma(G_2)$ it follows that $G_2 \in \mathfrak{X}$.*

Problem 1. (*a*) *Let Γ be a graph function. Describe all group classes (formations, Fitting classes, Schunk classes) that are recognizable by Γ.*

(*b*) *Let $\mathfrak{X}$ be a class of groups (formation, Fitting class, Schunk class). Find graph functions Γ that recognize $\mathfrak{X}$.*

T. Hawkes [4] in 1968 considered a directed graph of a group G whose set of vertices is $\pi(G)$ and (p,q) is an edge if and only if $q \in \pi(G/O_{p',p}(G))$. In particular he showed that a group G has a Sylow tower for some linear order ϕ if and only if its graph has not got circuits. We shall call this graph Hawkes graph and will denote it $\Gamma_H(G)$.

Theorem 1. *Let $\mathfrak{F}$ be a formation of groups. Then $\mathfrak{F}$ is recognized by Γ_H if and only if $\mathfrak{F} = LF(f)$ is a local formation where f is formation function defined as follows: $f(p) = \mathfrak{G}_{f(p)}$ if $p \in \pi(\mathfrak{F})$ and $f(p) = \emptyset$ otherwise.*

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Generalized supersoluble finite groups and mutually permutable products

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Only finite groups are considered. In the paper [1] V. A. Vedernikov introduced the notion of c-supersoluble group. Recall that a group G is called c-supersoluble if G has a chief series whose chief factors are simple. In [2] A.F. Vasil'ev and T. I. Vasil'eva continued studying of c-supersoluble groups using the method of composition satellites. D. Robinson established the structural properties of c-supersoluble groups in the paper [3].

In [4] authors introduced the notion of Jc-supersoluble group that is local analogue of c-supersoluble group. Let J is a some class (possibly empty) of simple groups. We say that a group G is a J-group if the set $\mathcal{K}_G$ of all composition factors of G is contained in J. Group G is called Jc-supersoluble if any chief J-factor of G is a simple group. A group G is called quasinilpotent (J-quasinilpotent) if for every chief factor (J-factor) H/K of G and every $x \in G$, x induces an inner automorphism on H/K.

In the [5] some properties of the products of normal Jc-supersoluble subgroups have been established. In this report we studied the mutually permutable products of Jc-supersoluble groups. Recall [6, p. 149] that group $G = HK$ is called the product of mutually permutable subgroups H and K, if H permutes with every subgroups of K and K permutes with every subgroups of H.

Theorem 1. *Let the group $G = HK$ be the product of the mutually permutable subgroups H and K. If G is an Jc-supersoluble group, then H and K are both Jc-supersoluble groups.*

Theorem 2. *Let the group $G = HK$ be the product of the mutually permutable subgroups H and K. If H is an Jc-supersoluble group and K is J-quasinilpotent, then G is an Jc-supersoluble group.*

Theorem 3. *Let the group $G = HK$ be the product of the mutually permutable subgroups H and K. If H and K are Jc-supersoluble groups and G', the derived subgroup of G, is J-quasinilpotent, then G is an Jc-supersoluble group.*

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On the pronormality and strong pronormality of Hall subgroups

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Throughout a set of primes is denoted by π. A subgroup H of G is called a *π-Hall subgroup*, if H is a π-group (i.e. all its prime divisors are in π), while the index of H is not divisible by primes from π. A subgroup is said to be a *Hall subgroup* if it is a π-Hall subgroup for some set of primes π. A subgroup H of G is called *pronormal*, if H and H^g are conjugate in $\langle H, H^g\rangle$ for every $g \in G$.

In Kourovka Notebook recorded the next problem [1, 18.32]: is every Hall subgroup of a finite group pronormal in its normal closure? The negative solution gives by the following

Theorem. *Let a set of primes π be such that*

(1) *there exists a simple group X which contains more than one class of conjugated π-Hall subgroups;*

(2) *there exists a simple group Y such that it contains a π-Hall subgroup which is not equal to self normalizer in Y.*

Thus in the regular wreath product $G = X \wr Y$ exists a not pronormal π-Hall subgroup, normal closure of which is equal to G.

For example, set $\{2, 3\}$ satisfies theorem conditions: group $X = L_3(2)$ contains two classes of conjugated $\{2, 3\}$-Hall subgroups and group $Y = L_2(16)$ contains $\{2, 3\}$-Hall subgroup which is not equal to self normalizer in Y.

A subgroup H of G is called *strongly pronormal*, if, for each $K \leq H$ and every $g \in G$, the subgroup K^g is conjugate with a subgroup of H (but not necessary with K) by an element from $\langle H, K^g\rangle$.

Also a negative solution of the problem [1, 17.45(б)] issue was obtained: in a finite simple group, are Hall subgroups always strongly pronormal?

More specifically, it was shown that $S_{10}(7)$ contains a $\{2, 3\}$-Hall subgroup, which is not strongly pronormal. Note that there are not known examples of pronormal Hall subgroups which are not strongly pronormal before.

The work is supported by Russian Science Foundation (project 14-21-00065).

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Perfect k-colorings of infinite circulant graphs with a continuous set of distances

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Consider an infinite graph $Ci_\infty(d_1, d_2, d_3, ..., d_n)$, whose set of vertices is the set of integers, and two vertices are adjacent if they are on the distance $d \in \{d_1, d_2, d_3, ..., d_n\}$. Let us call it an *infinite circulant graph*. Also we consider a finite graph $Ci_t(d_1, d_2, d_3, ..., d_n)$ with the set of vertices coinciding with the set Z_t and for each vertex v the multiset of incident edges is $\{(v, v + d_i \text{ mod t})|i = 1, 2, ..., n\}$. There is a natural homomorphism from the set of vertices of the graph $Ci_\infty(d_1, d_2, d_3, ..., d_n)$ on the set of vertices of the graph $Ci_t(d_1, d_2, d_3, ..., d_n)$ corresponding with the homomorphism from Z to Z_t.

Let k be a positive integer. A *k-coloring* of vertices of a graph $G = (V, E)$ is a map $\varphi : V \to \{1, 2, ..., k\}$. If $\varphi(v) = s$ for some vertex v, then s is the *color* of v.

A k-coloring of vertices is called *perfect*, if for each $i, j = 1, 2, ..., k$ are not necessarily different there is an uniquely defined non-negative integer α_{ij} which is equal to the number of vertices of the color j in the neighborhood of each vertex of the color i. The *period* T of a coloring is a sequence $\gamma_1\gamma_2...\gamma_r$, where $\gamma_i = \varphi(v_{m+i})$ for some number m, and $\varphi(v_l) = \varphi(v_{l+jr})$ for every l and j. The number r is the *length* of the period T. It is clear that the coloring of a regular graph is uniquely defined by its period.

Perfect 2-colorings of circulant graphs are considered in [1, 2]. We are interested in so-called circulant graphs with a continuous set of distances, i.e. in ones with the property $d_i = i, i = 1, 2, 3, ..., n$. The fool description of 2-colorings of graphs $Ci_\infty(n) = Ci_\infty(1, 2, ..., n)$ for an arbitrary positive integer n is given in [2]. A description of colorings with k colors for $k \geq 3$ presents severe difficulties, in particular, the natural homomorphism from n-dimensional grid Z^n on $Ci_\infty(n)$ shows that the problem is rather complicate.

Here we present the main result:

Theorem Let k, n be positive integers. The set of perfect colorings of a graph $Ci_\infty(n)$ contains all perfect colorings of graphs $Ci_t(n)$ for $t = 2n, 2n + 1, 2n + 2$ and the following ones:
1. $123...k$;
2. $123...(k-1)k(k-1)...32$;
3. $123...(k-1)kk(k-1)...32$;
4. $123...(k-1)kk(k-1)...321$.

It should be noted that last four colorings in the theorem are perfect for every n.

We conjecture that there are no other perfect colorings of the $Ci_\infty(n)$.

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On Waterman's lattices

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By finite lattice L can be determined ordinary graph $\langle L, \sim \rangle$, where $a \sim b$ means that a covers b or b covers a in the lattice L. Obviously, $Aut\langle L, \sim \rangle \supseteq Aut\langle L, \succ \rangle$. In the monograph of G. Birkhoff [1] posed the problem №6: "Determine all finite lattices in which every graph–automorphism is a lattice–automorphis" (A. G. Waterman). We call such finite lattices is Waterman's lattices.

In [2] proved a theorem that every finite lattice is embeddable into the Waterman's lattice.

We prove the following

Theorem. *Any finite lattice is a homomorphic image of a Waterman's lattice.*

These theorems indicate on the universalism and complexity of the Waterman's lattice class.

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On Some Variants of the Post Correspondence Problem

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The Post correspondence problem (PCP) is well known and one of the most useful undecidable problems [1, 2]. The undecidability of PCP was shown in [3]. A large number of variants of the problem have been considered.

Some variants of PCP are decidable. In particular, we can mention Marked PCP [4], PCP for 2 rules [5], PCP over the unary alphabet [6], silly Post correspondence problem (SPCP) [2], Post embedding problem [7], and regular Post embedding problem [7]. Also, there are a number of polynomial formulations of PCP with bounded length of the word [6, 8–11].

It should be noted that SPCP is one of the simplest variants of PCP. In particular, SPCP can be solved in linear time. However, for group and commutative alphabets, we obtain the following results.

Theorem 1. SPCP *is* **NP***-complete for commutative alphabet and bounded length of the word.*

Theorem 2. SPCP *is* **NP***-complete for group alphabet and bounded length of the word.*

Theorem 3. SPCP *is undecidable for group alphabet and 5 rules.*

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Group Shunkov, saturated groups $L_2(p^n), U_3(2^n)$

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Arbitrary group is called a Shunkov group, if every cross section by a finite subgroup of any pair of conjugate elements of prime order generates a finite subgroup. We emphasize that a Shunkov group, generated by elements of finite order, is not required to be periodic. Examples of such mixed groups already exist in the class of soluble groups [1]. Therefore, Shunkov groups pressing question about the locations of its elements of finite order, in particular, are they a characteristic subgroup of $T(G)$ — periodic part? Under the periodic part of $T(G)$ of a group G is the subgroup generated by all elements of finite order in G , provided that it is periodic.

In [2] considered groups Shunkov, saturated groups $L_2(p^n)$, $Sz(2^{2m+1})$. s shown that it has a periodic part, which, is isomorphic to either $L_2(P)$, or $Sz(Q)$ for suitable locally of finite fields P and Q.

In the present work, the study groups Shunkov, saturated groups $L_2(p^n)$, $U_3(2^n)$.

Obtained the following result.

Theorem. *The group Shunkov saturated with groups $L_2(p^n)$, $U_3(2^n)$ has a periodic part $T(G)$, is isomorphic to either $L_2(P)$, or $U_3(Q)$, where P and Q - suitable locally finite field.*

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On some subgroups of finite products of generalized nilpotent groups

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All groups considered are finite. Let a group $G = AB$ be a product of two its subgroups A and B. A subgroup H of $G = AB$ is called prefactorized if $H = (A \cap H)(B \cap H)$, it is called factorized [1] if, in addition, H contains the intersection $A \cap B$. For a saturated formation Heineken [2], for a Schunck class $\mathfrak{X}$ Amberg and Höfling [3] investigated prefactorized and factorized $\mathfrak{X}$-maximal subgroups (in particular $\mathfrak{X}$-projectors) of the group $G = AB$ with nilpotent subgroups A and B (see [4, 3.2.20, 3.2.22]).

We use notations and definitions from [5], [6]. Let π be a set of primes and π' the complement to π in the set of all primes. A group G is called π-decomposable if $G = G_\pi \times G_{\pi'}$ and a Hall π-subgroup G_π is nilpotent. The set of distinct primes dividing $|G|$ is denoted by $\pi(G)$. A non-empty homomorph $\mathfrak{X}$ is a Schunck class if any group G, all of whose primitive factor groups are in $\mathfrak{X}$, is itself in $\mathfrak{X}$. If $\mathfrak{H}$ and $\mathfrak{X}$ are classes of groups then $\mathfrak{H}\mathfrak{X} = (G|G$ has a normal subgroup $N \in \mathfrak{H}$ with $G/N \in \mathfrak{X})$. $\mathfrak{G}_{\pi'}$ denotes the class of all π'-groups.

Theorem. *Let $\mathfrak{X}$ be a class of groups and $\mathfrak{X} = \mathfrak{G}_{\pi'}\mathfrak{X}$. Let G be a π-soluble group and $G = AB$ be a product of two π-decomposable subgroups A and B.*

1) *If $\mathfrak{X}$ is a Schunck class such that $\pi(A) \cap \pi(B) \subseteq Char(\mathfrak{X})$, then every $\mathfrak{X}$-maximal subgroup of G has a factorized conjugate.*

2) *If $\mathfrak{X}$ is a saturated formation, then every $\mathfrak{X}$-maximal subgroup of G has a prefactorized conjugate.*

Recall that a subgroup H of a group G is an $\mathfrak{X}$-projector if HN/N is $\mathfrak{X}$-maximal in G/N for every normal subgroup N of G. If $\mathfrak{X}$ is a Schunck class and $\mathfrak{X} = \mathfrak{G}_{\pi'}\mathfrak{X}$ then every π-soluble group G has an $\mathfrak{X}$-projector and any two $\mathfrak{X}$-projectors of G are conjugate [7].

Corollary. *Let $\mathfrak{X}$ be a class of groups and $\mathfrak{X} = \mathfrak{G}_{\pi'}\mathfrak{X}$. Let G be a π-soluble group and $G = AB$ be a product of two π-decomposable subgroups A and B.*

1) *If $\mathfrak{X}$ is a Schunck class such that $\pi(A) \cap \pi(B) \subseteq Char(\mathfrak{X})$, then G has a unique factorized $\mathfrak{X}$-projector.*

2) *If $\mathfrak{X}$ is a saturated formation, then G has a unique prefactorized $\mathfrak{X}$-projector.*

The example 1 [3] shows that the condition $\pi(A) \cap \pi(B) \subseteq Char(\mathfrak{X})$ of theorem can not be discarded.

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Groups, saturated with unitary groups of dimension three.

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Group G is saturated with a set of groups X, if every finite subgroup K of G is contained in a subgroup of G, which is isomorphic to a member of X [3].

Article [1] gives a description of periodic groups saturated by groups from a set $\mathfrak{N} =\{U_3(2^n)|n$ – arbitrary positive integer$\}$. It was shown in [2] that a periodic Shunkov group, saturated by groups from a group set $\mathfrak{M} =\{\ U_3(p^m)|\ p$ – an arbitrary prime number, n – arbitrary positive integer$\}$, is isomorphic to $U_3(Q)$, where Q – is a suitable locally-finite field. The current work continues the investigations in that direction. Hereinafter, a symbol e will stand for the identity element of the group. The following results were obtained:

Theorem 1. *Let a periodic group G be saturated by groups from the set $\mathfrak{M}$ and S is the Sylow 2-subgroup of G takes one of the following forms:*

1. $S = \langle a^{2^n} = v^2 = 1, a^v = a^{2^{n-1}-1}\rangle$ – a semi-dihedral group.
2. $S = \langle a, w|a^{2^n} = b^{2^n} = w^2 = e, a^w = b, ab = ba\rangle$ – a wreath group.
3. S – is isomorphic to Sylow 2-subgroup $U_3(2^n)$.
4. S – is an infinite 2-group with a period of 4, nilpotency level equal 2, $S' = Z(S) = \Phi(S) = \Omega_1(S)$.
5. $S = (A \times B) \leftthreetimes \langle w\rangle$, where A – is an infinite locally-cyclic 2-group, $w^2 = e$, and $A^w = B$.
6. $S = AD$, where D is a finite subgroup of group S containing no wreath groups of order higher than 8, A – is an infinite locally-cyclic 2-group.

Theorem 2. *Shunkov group G, saturated with groups from the set $\mathfrak{M}$, has a periodic part $T(G)$, which is isomorphic to the group $U_3(Q)$, where Q is a suitable locally-finite field.*

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Special elements of the lattice of epigroup varieties

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A semigroup S is called an *epigroup* if, for any element x of S, some power of x lies in some subgroup of S. On an epigroup, a natural unary operation named *pseudoinversion* may be defined (see [1, 2], for instance). This allows us to consider varieties of epigroups as algebras with the operations of multiplication and pseudoinversion.

We continue an examination of special elements of the lattice **EPI** of all epigroup varieties started in [3].

An element x of a lattice $\langle L; \vee, \wedge \rangle$ is called *modular* if, for all $y, z \in L$, $(x \vee y) \wedge z = (x \wedge z) \vee y$ whenever $y \leq z$; *lower-modular* if, for all $y, z \in L$, $x \vee (y \wedge z) = y \wedge (x \vee z)$ whenever $x \leq y$; *distributive* if $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$ for all $y, z \in L$; *standard* if $(x \vee y) \wedge z = (x \wedge z) \vee (y \wedge z)$ for all $y, z \in L$; *neutral* if, for all $y, z \in L$, the sublattice of L generated by x, y and z is distributive. *Upper-modular*, *codistributive* and *costandard* elements are defined dually to lower-modular, distributive and standard ones respectively.

Neutral, modular and upper-modular elements of the lattice **EPI** are considered in [3]. Here we investigate lower-modular, costandard and codistributive elements of **EPI**.

Put $\mathcal{ZM} = \mathrm{var}\{xy = 0\}$ and $\mathcal{SL} = \mathrm{var}\{x^2 = x, xy = yx\}$. We denote by $\mathcal{T}$ the trivial epigroup variety.

Theorem 1. *An epigroup variety $\mathcal{V}$ is a costandard element of the lattice* **EPI** *if and only if $\mathcal{V}$ is one of the varieties $\mathcal{T}$, $\mathcal{SL}$, $\mathcal{ZM}$ or $\mathcal{SL} \vee \mathcal{ZM}$.*

Recall that a variety is called 0-*reduced* if it may be given by identities of the form $w = 0$ only.

Theorem 2. *An epigroup variety $\mathcal{V}$ is a lower-modular element of the lattice* **EPI** *if and only if $\mathcal{V} = \mathcal{M} \vee \mathcal{N}$ where $\mathcal{M}$ is one of the varieties $\mathcal{T}$ or $\mathcal{SL}$ and $\mathcal{N}$ is a 0-reduced variety.*

Theorems 1 and 2 together with results of [3] imply that an element of **EPI** is costandard if and only if it is neutral, is modular whenever it is lower-modular, and is distributive if and only if it is standard.

An epigroup variety is called *strongly permutative* if it satisfies an identity of the type $x_1 x_2 \ldots x_n = x_{1\pi} x_{2\pi} \ldots x_{n\pi}$ where π is a permutation on the set $\{1, 2, \ldots, n\}$ with $1 \neq 1\pi$ and $n \neq n\pi$.

Theorem 3. *A strongly permutative epigroup variety $\mathcal{V}$ is a codistributive element of the lattice* **EPI** *if and only if $\mathcal{V} = \mathcal{G} \vee \mathcal{X}$ where $\mathcal{G}$ is a variety of Abelian groups and $\mathcal{X}$ is one of the varieties $\mathcal{T}$, $\mathcal{SL}$, $\mathcal{ZM}$ or $\mathcal{SL} \vee \mathcal{ZM}$.*

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Minimal generating systems and properties of Sylow 2-subgroup of alternating group

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The aim of this paper is to research the structure of Sylow 2-subgroups and to construct a minimal generating system for such subgroups. Case of Sylow subgroup where $p = 2$ is very special because it admits odd permutations, this case was not investigated in [1, 2]. There was a mistake in a statement about irreducebility that system of $k + 1$ elements for $Syl_2(A_{2^k})$ which was in abstract [3] in 2015 year. All undeclared terms are from [4]. A minimal system of generators for a Sylow subgroup of A_n was found.

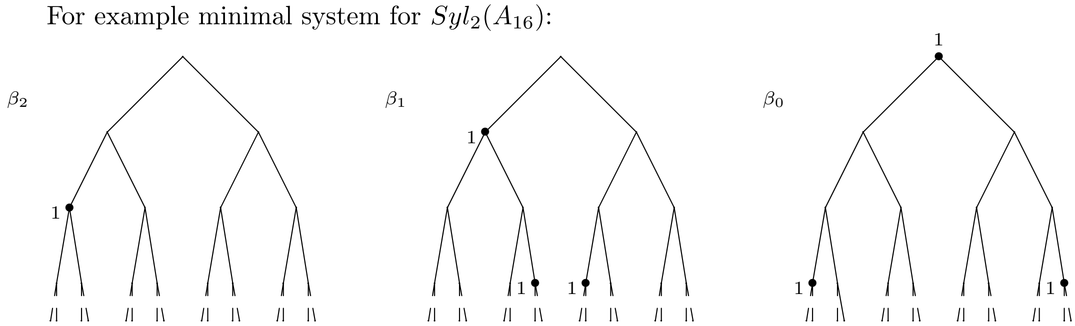
Let's denote by T_{k+1} a regular binary tree labeled by vertex. If the state in the vertex is non-trivial, then its label is 1, in other case it is 0. We denote by $v_{j,i}$ the vertex of L_j, which has the number i. An automorphism of T_{k+1} with non-trivial state in v_{1,i_1}, ... ,v_{1,i_j}, v_{2,j_2}, ... ,v_{k,k_m} is denoted by $\beta_{l_1,(i_1,\dots i_J);l_1(i_1,\dots i_J);\dots;l_{k-1}(i_1,\dots i_J)}$ where the index l_i is the number of level with non-trivial state. In parentheses after this numbers we denote a cortege of vertices of this level, where the non-trivial states in this automorphism are present. Denote by τ the automorphism, which has a non-trivial vertex permutation only in the first and the last vertices $v_{k,1}$ and $v_{k,2^k}$ of the last level L_k.

Lemma 1. *The set of elements from subgroup of $AutT_k$: $\alpha_{0,(1)}$, $\alpha_{1,(1)}$, $\alpha_{2,(1)}$, $\alpha_{k-2,(1)}$, τ, is system of generators for $Syl_2(A_{2^k})$.*

Lemma 2. *Orders of groups $\langle\alpha_{0,(1)}, \alpha_{1,(1)}, \alpha_{2,(1)}, \alpha_{k-2,(1)}, \tau\rangle$ and $Syl_2(A_{2^k})$ are equal to 2^{2^k-2}.*

Main Theorem. *The set of elements from subgroup of $AutT_k$ $\beta_{0,(1);k,(1,2^k)}$, $\beta_{1,(1);k,(2^{k-1},2^{k-1}+1)}$, $\beta_{2,(1)}$, ... ,$\beta_{k-2,(1)}$ is minimal generators for a Sylow 2-subgroup of A_{2^k}.*

For example minimal system for $Syl_2(A_{16})$:



It was proved that the structure of Sylow 2-subgroup of A_{2^k} is the following: $\wr_{i=1}^{k-1} C_2 \ltimes \prod_{i=1}^{2^{k-1}-1} C_2$, where we take C_2 as group of action on two elements and action is faithful.

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On complements for $\mathfrak{F}$-residuals in finite groups

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In the theory of groups are well known results on the complementarity of an $\mathfrak{F}$-residual $G^{\mathfrak{F}}$ in a finite group G where $\mathfrak{F}$ is a local formation (see, for example, [1]). Using the properties of $\mathfrak{F}$-normalizers of G we obtain new results on the complementarity of $G^{\mathfrak{F}}$ by $\mathfrak{F}$-normalizers of the group G where $\mathfrak{F}$ is an ω-local Fitting formation and $\omega \subseteq \pi(\mathfrak{F})$.

We consider only finite groups. Not listed designations and definitions can be found in [1]. Let ω be a non-empty subset of the set of all primes $\mathbb{P}$, $f : \omega \cup \{\omega'\} \to \{$ formations of groups $\}$ is an ωF-function. A formation $\mathfrak{F} = (G : G/O_\omega(G) \in f(\omega')$ and $G/F_p(G) \in f(p)$ for all $p \in \omega \cap \pi(G))$ is called an ω-local formation with the ω-satellite f. Following [2] (see definition 2.6.1 [2]) we state the following definitions.

Definition 1. Let $\mathfrak{F}$ be a non-empty formation. A normal subgroup R of the group G is called an $\mathfrak{F}$-limited normal subgroup of G if $R \leq G^{\mathfrak{F}}$ and $R/R \cap \Phi(G)$ is a chief factor of the group G. A maximal subgroup M of G is called $\mathfrak{F}$-critical in G if $G = MR$ for some $\mathfrak{F}$-limited normal subgroup R of G.

Definition 2. Let $\mathfrak{F}$ be a non-empty ω-local formation. A subgroup H of the group G is called an $\mathfrak{F}$-normalizer of G if $H/\Phi(H) \cap O_{\omega'}(H) \in \mathfrak{F}$ and there exists a maximal chain $H = H_t \subset H_{t-1} \subset \cdots \subset H_1 \subset H_0 = G$ where H_i is an $\mathfrak{F}$-critical subgroup of H_{i-1} for each $i = 1, 2, \ldots, t$ and $0 \leq t$.

Theorem 1. *Let $\mathfrak{F}$ be a non-empty ω-local formation and let G be a group. Then there exists at least one $\mathfrak{F}$-normalizer H of the group G and $G = G^{\mathfrak{F}}H$.*

Theorem 2. *Let $\mathfrak{F}$ be a non-empty ω-local Fitting formation and let $G = A_1A_2 \cdots A_n$ be a group where A_i is a subnormal subgroup of G for each $i = 1, 2, \ldots, n$ and $\omega \subseteq \pi = \pi(\mathfrak{F})$. If a $\mathfrak{F}$-residual of A_i is ω-soluble and for every $p \in \omega$ Sylow p-subgroups of $A_i^{\mathfrak{F}}$ is abelian for each $i = 1, 2, \ldots, n$ then every $\mathfrak{F}$-normalizer of G is an ω-complement for $G^{\mathfrak{F}}$ in G.*

Corollary 1. *Let $\mathfrak{F}$ be a local non-empty Fitting formation and let $G = A_1A_2 \cdots A_n$ be a group where A_i is a subnormal subgroup of G for each $i = 1, 2, \ldots, n$. If an $\mathfrak{F}$-residual $A_i^{\mathfrak{F}}$ is $\pi(\mathfrak{F})$-soluble for every $i = 1, 2, \ldots, n$ and its Sylow p-subgroups are abelian for all $p \in \pi(\mathfrak{F})$ then each $\mathfrak{F}$-normalizer of G is the complement for $\mathfrak{F}$-residual $G^{\mathfrak{F}}$ in G.*

Corollary 2. *Let $\mathfrak{F}$ be a local non-empty Fitting formation and let $G = A_1A_2 \cdots A_n$ be a group where A_i is a subnormal subgroup of G for each $i = 1, 2, \ldots, n$. If $\mathfrak{F}$-residual $A_i^{\mathfrak{F}}$ is abelian for every $i = 1, 2, \ldots, n$ then each $\mathfrak{F}$-normalizer of G is the complement for $\mathfrak{F}$-residual $G^{\mathfrak{F}}$ in G.*

Theorem 3 *Let $\mathfrak{F}$ be a non-empty ω-local formation, let G be a group and let ω_1 be a set of all primes $p \in \omega$ for which $G^{\mathfrak{F}}$ has an abelian Sylow p-subgroup. Then $G^{\mathfrak{F}}$ has an ω_1-complement in any extension of G.*

Theorem 4 *Let $\mathfrak{F}$ be a non-empty ω-local formations, let Γ be an extension of the group G and let $\omega_1 = \{p \in \mathbb{P} |\ p$ divides $(|\ \Gamma : G^{\mathfrak{F}}\ |, |\ G^{\mathfrak{F}}\ |)\}$. If $\omega_1 \subseteq \omega$ and a Sylow p-subgroup of $G^{\mathfrak{F}}$ is abelian for each $p \in \omega_1$, then $G^{\mathfrak{F}}$ has a complement in the group Γ.*

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The eigenfunctions with the minimum support of the cubic distance-regular graphs

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Let $G = (V, E)$ be an undirected graph without loops and multiple edges with the vertex set $V = \{1, 2, \ldots, n\}$ and the edge set E. G is *regular* if each vertex has the same number k of the neighbours. The parameter k is called the *degree* of the graph. For any vertices $v, u \in V$ the *distance* $d(v, u)$ is the number of edges in the shortest path that connects them. By $G_i(v)$ we denote the set of the vertices that are at distance i from v. A connected graph G is called *distance-regular* if it is regular of degree k and for any two vertices $v, u \in V$ at distance $i = d(v, u)$, there are precisely c_i neighbours of u in $G_{i-1}(v)$ and b_i neighbours of u in $G_{i+1}(v)$. The numbers $b_i, c_i, a_i = k - b_i - c_i$ are called the *intersection numbers* of G.

Consider the adjacency matrix A of order n, defined as following:

$$A_{ij} = \begin{cases} 1, & \text{when } ij \in E \\ 0, & \text{when } ij \notin E \end{cases}$$

For a matrix A let $\Lambda = \{\lambda_1, \ldots, \lambda_t\}$ be the set of its eigenvalues. If $f = (f_1, \ldots, f_n)$ is a function on the graph vertices that satisfies the equation $Af = \lambda f$, we call it an *eigenfunction* of the graph G corresponding to the eigenvalue λ. The support $supp(f)$ of the function f is the set of its non-zero coordinates, i.e. $supp(f) = \{i \mid f_i \neq 0\}$. We are interested in finding the eigenfunctions with the supports of minimum cardinality.

In the current work we study the distance-regular graphs of the degree $k = 3$. It is known [1] that up to isomorphism there are only 13 of them: K_4, $K_{3,3}$, the Petersen graph, the cube, the Heawood graph, the Pappus graph, the Coxeter graph, the Tutte-Coxeter graph, the dodecahedron, the Desargues graph, the Foster graph, the Tutte 12-cage, the Biggs-Smith graph. For all of them, except for the last two graphs, we found the cardinalities of the minimum supports of the eigenfunctions over the field $\mathbb{R}$ and classified their structures for all the eigenvalues.

This research was funded by the Russian Science Foundation (grant No 14-11-00555).

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On arc-transitive distance-regular covers of complete graphs related to $SU_3(q)$

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In 1991, P. J. Cameron has discovered a family of arc-transitive distance-regular covers of complete graphs, which are obtained by the following construction proposed in [3, p.90]. Let E be the quadratic extension of the finite field F of q elements. Denote by V the 3-dimensional vector space over E equipped with a non-degenerate Hermitian form B. Let U be a subgroup of E^* of index r. Let Ψ_r be the graph on the set of U-orbits on the isotropic vectors of V with two vertices vU and wU being adjacent if and only if $B(v, w) = 1$. By [3, Proposition 5.1 (iv)] Ψ_r is distance-regular (with intersection array $\{q^3, (r-1)(q^2-1)(q+1)/r, 1; 1, (q^2-1)(q+1)/r, q^3\}$) if and only if either q is even and r divides $q+1$, or q is odd and r divides $(q+1)/2$. The question naturally arises whether this family comprises (up to isomorphism) all distance-regular covers of complete graphs with the antipodality index dividing $q+1$, which possess an arc-transitive automorphism group, isomorphic to $SU_3(q)$. As we will show below, it turns out, that the answer is negative.

Let $G = SU_3(q)$ denote the special unitary group on V and put $K = G_{\langle e_1\rangle,\langle e_2\rangle}$, where e_1 and e_2 are two non-collinear isotropic vectors of V. Take P to be the subgroup of K of order $q-1$, and let S be the subgroup of $G_{\langle e_1\rangle}$ of order q^3. Put $H = SP$. Assume that g is a 2-element of G interchanging $\langle e_1\rangle$ with $\langle e_2\rangle$ such that $g^2 \in H$. Let $\Gamma(G, H, HgH)$ denote the graph with vertex set $\{Hx \mid x \in G\}$ whose edges are the pairs $\{Hx, Hy\}$ such that $xy^{-1} \in HgH$.

Theorem. *If q is odd, then $\Gamma(G, H, HgH)$ is distance-regular if and only if g is an element of order 4, while if q is even, then g is an involution and $\Gamma(G, H, HgH)$ is a distance-regular graph isomorphic to Ψ_{q+1}. In both cases, the resulting distance-regular graph has intersection array $\{q^3, q(q^2-1), 1; 1, q^2-1, q^3\}$, does not depend on the choice of the element g (of the given order) and admits distance-regular quotients with intersection array $\{q^3, (i-1)(q^2-1)(q+1)/i, 1; 1, (q^2-1)(q+1)/i, q^3\}$ for each proper divisor i of $q+1$.*

Remark. *Assume that q is odd and let g be of order 4. Distance-regularity of $\Gamma(G, H, HgH)$ appear to be first shown in the course of this work. Note that if γ is an element of E^* such that $\gamma^q = -\gamma$ and $U = F^*$, then $\Gamma(G, H, HgH)$ is isomorphic to the graph Φ on the set of U-orbits on the isotropic vectors of V with two vertices vU and wU being adjacent if and only if $B(v, w) \in U\gamma$. The construction of the graph Φ fits in the construction described in [2, Proposition 12.5.4], which generalizes the Cameron construction. However, the case $r = q+1$ for an odd q has not been completely considered in [2]. Note also, that if in definition of Φ we assume $\gamma \in U$ instead of the condition $\gamma^q = -\gamma$, then we get $\Phi \simeq \Psi_{q+1} \simeq \Gamma(G, H, HgH)$ for an involution g.*

This work was supported by Russian Foundation for Basic Research (research project No. 14-01-31298).

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On finite groups with submodular Sylow subgroups

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Throughout these abstracts, all groups are finite. Recall that a subgroup M of a group G is called modular in G, if the following hold:
1) $\langle X, M \cap Z\rangle = \langle X, M\rangle \cap Z$ for all $X \leq G, Z \leq G$ such that $X \leq Z$, and
2) $\langle M, Y \cap Z\rangle = \langle M, Y\rangle \cap Z$ for all $Y \leq G, Z \leq G$ such that $M \leq Z$.
Note that a modular subgroup is a modular element (in the sense of Kurosh [1, Chapter 2, p. 43]) of a lattice of all subgroups of a group. Properties of modular subgroups were studied in the book [1]. Groups with all subgroups are modular were studied by R. Schmidt [1], [2] and I. Zimmermann [3]. By parity of reasoning with subnormal subgroup, in [3] the notion of a submodular subgroup was introduced.

Definition [3]. *A subgroup H of a group G is called submodular in G, if there exists a chain of subgroups $H = H_0 \leq H_1 \leq \ldots \leq H_{s-1} \leq H_s = G$ such that H_{i-1} is a modular subgroup in H_i for $i = 1, \ldots, s$.*

It's well known that in a nilpotent group every Sylow subgroup is normal (subnormal). In the paper [3] groups with submodular subgroups were studied. In particular, it was proved that in a supersoluble group G every Sylow subgroup is submodular if and only if $G/F(G)$ is abelian of squarefree exponent. A criterion of the submodularity of Sylow subgroups in an arbitrary group was found.

We continue study of groups with submodular Sylow subgroups. A group we call strongly supersoluble and denote $s\mathfrak{U}$, if it is supersoluble and every Sylow subgroup is submodular in it. Denote $\mathfrak{V}$ a class of all abelian groups of exponent free from squares of primes; $sm\mathfrak{U} = (\ G \mid$ every Sylow subgroup of the group G is submodular in $G\ )$.

We obtained the following results:

Theorem 1. *Let G be a group. Then the following hold:*
1) *if $G \in sm\mathfrak{U}$ and $H \leq G$, then $H \in sm\mathfrak{U}$;*
2) *if $G \in sm\mathfrak{U}$ and $N \trianglelefteq G$, then $G/N \in sm\mathfrak{U}$;*
3) *if $N_i \trianglelefteq G$ and $G/N_i \in sm\mathfrak{U}$, $i = 1, 2$, then $G/N_1 \cap N_2 \in sm\mathfrak{U}$;*
4) *if $H_i \in sm\mathfrak{U}$, $H_i \trianglelefteq G$, $i = 1, 2$ and $H_1 \cap H_2 = 1$, then $H_1 \times H_2 \in sm\mathfrak{U}$;*
5) *if $G/\Phi(G) \in sm\mathfrak{U}$, then $G \in sm\mathfrak{U}$;*
6) *the class of groups $sm\mathfrak{U}$ is a hereditary saturated formation.*

Theorem 2. *The class of all groups with submodular Sylow subgroups is a local formation and has a local screen f such that $f(p) = (G \in \mathfrak{S} \mid \mathrm{Syl}(G) \subseteq \mathfrak{A}(p-1) \cap \mathfrak{V})$ for every prime p.*

Theorem 3. *Let G be a group. Then the following statements are equivalent:*
1) *every Sylow subgroup is submodular in G;*
2) *G is Ore dispersive and every its biprimary subgroup is strongly supersoluble;*
3) *every metanilpotent subgroup of G is strongly supersoluble.*

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About some products K-$\mathbb{P}$-subnormal subgroups of finite groups

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We consider only finite groups. In 1978 O. Kegel [1] proposed the concept of K-$\mathfrak{F}$-subnomal subgroup.

Let $\mathfrak{F}$ be a non-empty hereditary formation. A subgroup H of a group G is called K-$\mathfrak{F}$-subnormal ($\mathfrak{F}$-reachable [1]) subgroup of G (denoted H K-$\mathfrak{F}$-sn G), if there is a chain of subgroups $H = H_0 \subseteq H_1 \subseteq \cdots \subseteq H_n = G$ such that or $H_{i-1} \triangleleft H_i$, or $H_i^{\mathfrak{F}} \subseteq H_{i-1}$, for $i = 1, \ldots, n$.

In papers [2] and [3] A. F. Vasil'ev, T. I. Vasil'eva, V. N. Tyutyanov introduced the definitions of $\mathbb{P}$-subnormality and K-$\mathbb{P}$-subnormality for subgroups respectively.

Definition 1 [3]. *A subgroup H of group G is called* K-$\mathbb{P}$-*subnormal in G (denoted H* K-$\mathbb{P}$-*sn G), if there is a chain of subgroups $H = H_0 \subseteq H_1 \subseteq \cdots \subseteq H_n = G$ such that either H_{i-1} is normal in H_i or $|H_i : H_{i-1}|$ is prime for every $i = 1, \ldots, n$.*

Let $\mathfrak{U}$ be the formation of all supersoluble groups, then every K-$\mathfrak{U}$-subnormal subgroup of G is K-$\mathbb{P}$-subnormal in G. The converse assertion fails to hold in general.

In [3] authors studied the properties of products of groups $G = AB$ where A and B are K-$\mathbb{P}$-subnormal in G. In the present article we continue investigations of [3] in the case if a group G is the product of its pairwise permutable subgroups $G_1, G_2, \ldots, G_n$, ie $G = G_1G_2 \ldots G_n$ and $G_iG_j = G_jG_i$ for all integers i and j with $i, j \in \{1, 2, \ldots n\}$.

Definition 2 [3]. *A group G is called $\overline{\mathrm{w}}$-supersoluble if every Sylow subgroup of G is* K-$\mathbb{P}$-*subnormal in G.*

Theorem 1. *Let $G = G_1G_2 \ldots G_n$ be the product of its pairwise permutable Ore dispersive subgroups G_1 $G_2, \ldots, G_n$, subgroups G_i* K-$\mathbb{P}$-*sn G_iG_j and G_j* K-$\mathbb{P}$-*sn G_iG_j for each $i, j \in \{1, 2, \ldots n\}$. Then G is Ore dispersive.*

Theorem 2. *Let $G = G_1G_2 \ldots G_n$ be the product of its pairwise permutable nilpotent subgroups G_1 $G_2, \ldots, G_n$, subgroups G_i* K-$\mathbb{P}$-*sn G_iG_j and G_j* K-$\mathbb{P}$-*sn G_iG_j for each $i, j \in \{1, 2, \ldots n\}$. Then G is $\overline{\mathrm{w}}$-supersoluble.*

Recall [2] a generalized commutant of a group G is called the smallest normal subgroup N of G such that G/N is a group with abelian Sylow subgroups.

Theorem 3. *Let $G = G_1G_2 \cdots G_n$ be the product of its pairwise permutable $\overline{\mathrm{w}}$-supersoluble subgroups $G_1, G_2, \ldots, G_n$, subgroups G_i* K-$\mathbb{P}$-*sn G_iG_j and G_j* K-$\mathbb{P}$-*sn G_iG_j for each $i, j \in \{1, 2, \ldots n\}$. If the generalized commutant of group G is nilpotent, then G is $\overline{\mathrm{w}}$-supersoluble.*

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On finite groups generated by involutions

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The problem of classification of all finite 2-groups generated by three involutions appears to be difficult.

In [1] classification of metabelian groups with this condition was announced and also the list of such groups with elementary abelian commutator subgroups was presented.

In [2] was proved that finite 2-groups generated by three involutions of exponent 4 have order $\leq 2^{10}$ and they are classified.

In present communication we announce two theorems, the first of which is considered in the class of all finite groups.

Theorem 1. *For any finite group A generated by involutions there exists a finite group B generated by three involutions with a series of subgroups:*

$$1 \leq N \leq G \leq B, \text{ where}$$

$$N \trianglelefteq B, G/N \simeq A.$$

The proof of theorem 1 with some additional considerations implies theorem 2.

Theorem 2. *There exist finite 2-groups generated by three involutions of arbitrarily large derived length.*

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Upper-modular and related elements of the lattice of commutative semigroup varieties

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We denote by **SEM** the lattice all semigroup varieties and by **Com** the sublattice of **SEM** consisting of all commutative varieties. During last decade, about 15 articles devoted to examination of special elements of different types in these two lattices were appeared. The results obtained here are overviewed in the recent article [1]. Special elements of eight types were considered in the mentioned articles, namely neutral, standard, costandard, distributive, codistributive, modular, lower-modular and upper-modular elements (the definitions see in [1]). In the lattice **SEM**, neutral, standard, costandard, distributive or lower-modular elements are completely described, and a significant results concerning codistributive, modular or upper-modular elements were obtained. In the lattice **Com**, neutral, standard, distributive or lower-modular elements were completely determined, and a significant results about modular elements were proved. But there no any information about costandard, codistributive or upper-modular elements in **Com** up to the recent time. The following two theorems give a complete description of these elements.

Theorem 1. *For a commutative semigroup variety $\mathcal{V}$, the following are equivalent:*

a) *$\mathcal{V}$ is an upper-modular element in the lattice* **Com***;*

b) *$\mathcal{V}$ is a codistributive element in the lattice* **Com***;*

c) *one of the following holds:*

   (i) *$\mathcal{V}$ is the variety of all commutative semigroups;*

   (ii) *$\mathcal{V} = \mathcal{M} \vee \mathcal{N}$ where $\mathcal{M}$ is either the trivial variety $\mathcal{T}$ or the variety of semilattices $\mathcal{SL}$, and $\mathcal{N}$ is a commutative variety with the identities $x^2yz = 0$ and $x^2y = xy^2$;*

   (iii) *$\mathcal{V} = \mathcal{G} \vee \mathcal{M} \vee \mathcal{N}$ where $\mathcal{G}$ is a variety of periodic Abelian groups, $\mathcal{M}$ is one of the varieties $\mathcal{T}$, $\mathcal{SL}$ or $\mathrm{var}\{x^2 = x^3, xy = yx\}$, and $\mathcal{N}$ is a commutative variety with the identity $x^2y = 0$.*

Theorem 2. *A commutative semigroup variety $\mathcal{V}$ is a costandard element in the lattice* **Com** *if and only if one of the claims* (i) *or* (ii) *of Theorem* 1 *holds.*

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On intersections of nilpotent subgroups in finite groups with the socle isomorphic to $\Omega_8^+(2)$

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Let G is a finite group with the socle $Soc(G)$ isomorphic to $\Omega_8^+(2)$. Then (see [1]) $Out(\Omega_8^+(2)) \cong \Sigma_3$ and $Soc(G)$ contains a parabolic subgroup P such that P is normalized by an involution τ which induces the graph automorphism on $Soc(G)$ and Levi subgroup of P is isomorphic to $L_3(2)$.

For subgroups A and B of G, denote by $M_G(A,B)$ the set of minimal under the inclusion intersections $A \cap B^g$ where $g \in G$ and by $m_G(A,B)$ the set of minimal under the order elements from $M_G(A,B)$. Set $Min_G(A,B) = \langle M_G(A,B)\rangle$ and $min_G(A,B) = \langle m_G(A,B)\rangle$.

The following two theorems are proved.

Theorem 1. *Let G be a finite group with $Soc(G) \cong \Omega_8^+(2)$ and $S \in Syl_2(G)$. If $min_G(S,S) \neq 1$ then $G = Soc(G)\langle\tau\rangle$ and $min_G(S,S) = O_2(P)\langle\tau\rangle$.*

Theorem 2. *Let G be a finite group with $Soc(G) \cong \Omega_8^+(2)$, $S \in Syl_2(G)$, A and B be nilpotent subgroups of G. Then the following conditions are equivalent:*

(1) *$A \cap B^g \neq 1$ for any $g \in G$;*

(2) *$min_G(A,B) \neq 1$;*

(3) *$Min_G(A,B) \neq 1$;*

(4) *$G = Soc(G)\langle\tau\rangle$, A and B are conjugated to some subgroups A^g and B^h of S such that $A^g \cap B^h \geq min_G(S,S)$.*

This work was supported by Russian Science Foundation, project no. 15-11-10025.

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The International Conference and PhD Summer School «Groups and Graphs, Algorithms and Automata»

Yekaterinburg, August 09-15, 2015

Sunday August 9	Monday August 10	Tuesday August 11	Wednesday August 12	Thursday August 13	Friday August 14	Saturday August 15
9:00 – 12:00 Registration 12:30 – 13:30 Getting Ivolga 14:00 – 15:00 Lunch 15:30-15:45 Official Welcome 15:45 – 17:40 Plenary Talks: Special session dedicated to Professor Vitaly Baransky 15:45 – 16:00 Greeting to Professor Vitaly Baransky 16:00 – 16:50 Vitaly Baransky 16:50 – 17:40 Alexander Makhnev 17:45 – 19:00 Football 19:00 – 20:00 Dinner	8:30 – 9:45 Breakfast					
	10:00 – 13:00 Plenary Talks	10:00 – 13:00 Plenary Talks and Minicourses	10:00 – 13:15 Plenary Talks: Special session dedicated to Professor Vyacheslav Belonogov	10:00 – 13:00 Minicourses		
	10:00 – 10:50 Tatsuro Ito	10:00 – 10:50 Sergey Goryainov	10:00 – 10:15 Greeting to Professor Vyacheslav Belonogov 10:15 – 11:05 Vyacheslav Belonogov	10:00 – 10:50 Mikhail Volkov: Minicourse V, Lecture 1	10:00 – 10:50 Mikhail Volkov: Minicourse V, Lecture 2	10:00 – 10:50 Mikhail Volkov: Minicourse V, Lecture 3
	10:50 – 11:40 Jack Koolen	10:50 – 11:40 Evgeny Vdovin: Minicourse IV, Lecture 1	11:05 – 11:55 Lev Kazarin	10:50 – 11:40 Evgeny Vdovin: Minicourse IV, Lecture 2	10:50 – 11:40 Nadezhda Timofeeva: Minicourse III, Lecture 1	10:50 – 11:40 Nadezhda Timofeeva: Minicourse III, Lecture 2
	11:40 – 12:10 Coffee break		11:55 – 12:25 Coffee break	11:40 – 12:10 Coffee break		
	12:10 – 13:00 Vladislav Kabanov	12:10 – 13:00 Tomaž Pisanski: Minicourse II, Lecture 1	12:25 – 13:15 Bernhard Amberg	12:10 – 13:00 Tomaž Pisanski: Minicourse II, Lecture 2	12:10 – 13:00 Tomaž Pisanski: Minicourse II, Lecture 3	12:10 – 13:00 Tomaž Pisanski: Minicourse II, Lecture 4
	13:00 – 14:00 Lunch		13:15 – 14:15 Lunch	13:00 – 14:00 Lunch		
	14:30 – 16:10 Minicourses		14:30 – 17:30 Plenary Talks	14:30 – 16:10 Minicourses		14:30 – 15:20 Plenary Talks
	14:30 – 15:20 Dragan Marušič: Minicourse I, Lecture 1	14:30 – 15:20 Dragan Marušič: Minicourse I, Lecture 3	14:30 – 15:20 Anatoly Kondrat'ev	14:30 – 15:20 Dragan Marušič: Minicourse I, Lecture 5	14:30 – 15:20 Dragan Marušič: Minicourse I, Lecture 7	14:30 – 15:20 Vladimir Trofimov
	15:20 –16:10 Klavdija Kutnar: Minicourse I, Lecture 2	15:20 –16:10 Klavdija Kutnar: Minicourse I, Lecture 4	15:20 – 16:10 Natalia Maslova	15:20 –16:10 Klavdija Kutnar: Minicourse I, Lecture 6	15:20 –16:10 Klavdija Kutnar: Minicourse I, Lecture 8	15:30 – 15:50 Closing
	16:10 – 16:40 Coffee break					17:00 Leaving Ivolga
	16:40 – 19:00 Contributed talks		16:40 – 17:30 Vladimir Levchuk	16:40 – 19:00 Contributed talks		
	19:00 – 20:00 Dinner		17:45 – 18:00 Conference Photo	19:00 – 20:00 Dinner		
	20:00 – 22:00 Problem Solving / Sports		Conference Dinner	20:00 – 22:00 Problem Solving / Sports		

Graphs and Groups, Spectra and Symmetries

Akademgorodok, Novosibirsk, Russia, August, 15 – 28, 2016

Announcement

Sobolev Institute of Mathematics of Siberian Branch of Russian Academy of Sciences and Novosibirsk State University organize the International Conference and PhD-Magister Summer School "Graphs and Groups, Spectra and Symmetries" (G2S2). It will be held in Akademgorodok, Novosibirsk, Russia, August, 15 – 28, 2016.

Summer School Minicourses will be given by:

Alexander A. Ivanov, Imperial College, London, UK
Lih Hsing Hsu, Providence University, Taichung, Taiwan
Bojan Mohar, Simon Fraser University, Canada

Short descriptions of Minicourses

Minicourse 1: Y-groups via Majorana Theory

Lecturer: Alexander A. Ivanov, Department of Mathematics, Imperial College, London, UK

Syllabus: Motivated by an earlier observation by B. Fischer, around 1980 J. H. Conway conjectured that a specific Coxeter diagram Y_{443} together with a single additional (so-called "spider") relation form a presentation for the direct product of the largest sporadic simple group known as the Monster and a group of order 2. This conjecture was proved by S. P. Norton and the lecturer in 1990. It appears promising to revisit this subject through currently developing axiomatic approach to the Monster and its non-associative 196884-dimensional algebra, which goes under the name "Majorana Theory".

Minicourse 2: Another viewpoint of Euler graphs and Hamiltonian graphs

Lecturer: Lih Hsing Hsu, Distinguished Professor, Providence University, Taichung, Taiwan

Syllabus: It may appear that there is little left to do in regards to the study of the Hamiltonian property of vertex transitive graphs unless there is a major breakthrough on the famous Lovasz conjecture. However, if we extend the concept of the traditional Hamiltonian property to other Hamiltonicity properties, then there is still much left to explore. In this series of lectures, I will introduce some of these Hamiltonicity properties, namely fault tolerant Hamiltonian, spanning connectivity, and mutually independent Hamiltonicity.