Minicourse V: Synchronizing finite automata: a problem everyone can understand but nobody can solve (so far)

Lecturer:
Mikhail Volkov
Institute of Mathematics and Computer Science, Ural Federal University, Yekaterinburg, Russia

Most current mathematical research, since the 60s, is devoted to fancy situations: it brings solutions which nobody understands to questions nobody asked (quoted from Bernard Beauzamy in [1]). This provocative claim is certainly exaggerated but it does reflect a really serious problem: a communication barrier between mathematics (and exact science in general) and human common sense. The barrier results in a paradox: while the achievements of modern mathematics are widely used in many crucial aspects of everyday life, people tend to believe that today mathematicians do "abstract nonsense" of no use at all. In most cases it is indeed very difficult to explain to a non-mathematician what mathematicians work with and how their results can be applied in practice. Fortunately, there are some lucky exceptions, and one of them has been chosen as the present course's topic. It is devoted to a mathematical problem that was frequently asked by both theoreticians and practitioners in many areas of science and engineering. The problem, usually referred to as the synchronization problem, can be roughly described as the task of determining the simplest way to restore control over a device whose current state is not known - think of a satellite which loops around the Moon and cannot be controlled from the Earth while "behind" the Moon. While easy to understand and practically important, the synchronization problem turns out to be surprisingly hard to solve even for finite automata that constitute the simplest mathematical model of real-world devices. This combination of transparency, usefulness and unexpected hardness should, hopefully, make the course interesting for a wide audience.

Among other things, the course will present a recent major advance in the theory of synchronizing finite automata: Avraam Trahtman's proof of the so-called Road Coloring Conjecture by Adler, Goodwyn, and Weiss. The conjecture that admits a formulation in terms of recreational mathematics arose in symbolic dynamics and has important implications in coding theory. The proof is elementary in its essence but clever and enjoyable.

The course contains 3 lectures.

Reference

[1] B. Beauzamy, Real life Mathematics // Irish Math. Soc. Bull. 2002. Vol. 48. P. 43–46.