On products of groups which contain almost abelian subgroups

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Let the group $G = AB$ be the product of two subgroups A and B, i. e. $G = \{ab \mid a \in A, b \in B\}$. If A and B are abelian, then G is metabelian by a well-known theorem of N. Itô (see for instance [1]). This raises the question whether every group $G = AB$ with abelian-by-finite subgroups A and B is metabelian-by-finite ([1], Question 3), or at least soluble-by-finite. However, this seemingly simple question is very difficult to attack. A positive answer was previously given under additional requirements, for instance for linear groups G by Ya. Sysak and for residually finite groups G by J. Wilson, see [1]. Furthermore, N. S. Chernikov proved that every group $G = AB$ with central-by-finite subgroups A and B is soluble-by-finite (see [1]).

It is natural first to consider groups $G = AB$ where the two factors A and B have abelian subgroups with small index, in particular less or equal 2. In the talk some results obtained recently by Lev Kazarin, Yaroslav Sysak and myself in the case that enough involutions are present will be presented.

For example the following holds.

Theorem. Let the group $G = AB$ be the product of two subgroups A and B each of which is either abelian or generalized dihedral. Then G is soluble.

Here a group A is called generalized dihedral if it contains an abelian subgroup X of index 2 and an involution inverting every element of X. Clearly A is a semidirect product $A = X \rtimes < a >$ of the abelian group X with a group $< a >$ of order 2 such that $x^a = x^{-1}$ for every $x \in X$. Obviously dihedral groups and locally dihedral groups are also generalized dihedral.

Reference

[1] B. Amberg, S. Franciosi, F. de Giovanni, Products of Groups. The Clarendon Press, Oxford University Press: Oxford, 1992.