

The lattice of graphical partitions

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A *partition* is a sequence of nonnegative integers (the parts) in nonincreasing order (we will disregard trailing zeroes).

A *graphical partition* is a partition whose parts can be interpreted as the degrees of the vertices of some simple (undirected) graph.

We show that, for a given integer n ,

- all graphical partitions,
- all graphical partitions of lengths less than or equal to n ,
- all graphical partitions of length n

form the lattices GPL , $GPL(n)$, $GPLzf(n)$ ordered by dominance.

We show that the lattice GPL is a lower subsemilattice of the lattice NPL of all partitions ordered by dominance, but GPL is not a sublattice of the lattice NPL .

We establish that the set of all graphical partitions of $2m$ is an order ideal of the lattice of all partitions of $2m$. We find, for a given integer m , all maximal graphical partitions in the lattice of all partitions of $2m$.

We also present a new algorithm, which, for a given integer n , generates all graphical partitions of lengths less than or equal to n . Our algorithm can generate graphical partitions without generating any nongraphical partitions.

Reference

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