

On the pronormality of subgroups of odd indices in finite simple groups

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A subgroup H of a group G is said to be *pronormal* in G if H and H^g are conjugate in $\langle H, H^g \rangle$ for every $g \in G$.

In [1], the following conjecture was formulated.

Conjecture. *All subgroups of odd indices are pronormal in all finite simple groups.*

In [2], the following theorem is proved.

Theorem. *All subgroups of odd indices are pronormal in the following finite simple groups:*

- (1) A_n , where $n \geq 5$;
- (2) sporadic groups;
- (3) groups of Lie type over fields of characteristic 2;
- (4) $L_{2^n}(q)$;
- (5) $U_{2^n}(q)$;
- (6) $S_{2n}(q)$, where $q \not\equiv \pm 3 \pmod{8}$;
- (7) $O_n(q)$;
- (8) exceptional groups of Lie type not isomorphic to $E_6(q)$ or ${}^2E_6(q)$.

In this talk, we construct a counterexample to mentioned conjecture and discuss (in progress) a classification of finite simple groups in which all subgroups of odd indices are pronormal.

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References

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